The "Individuality" of a Quantum Event

Remarks on Whitehead's Epochal Theory of Time and Bohr's Framework of Complementarity

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1. Whitehead's Epochal Theory of Time: -Genetic Analysis and Coordinate Analysis-

The distinction between two modes of analysis of an actual occasion, i.e. genetic and coordinate, is fundamental in Whitehead's "epochal" theory of time. Genetic analysis divides the "concrescence" (the process of becoming concrete), and coordinate analysis divides the concrete (thing). The concrete is in its "satisfaction", but the concrescence is the passage from real potentiality to actuality. Both can be objects for analysis but under the different perspectives. Whitehead states:[i]

> Physical time makes its appearance in the 'coordinate' analysis of the satisfaction.' The actual entity is the enjoyment of a certain quantum of physical time. But the genetic process is not the temporal succession: such a view is exactly what is denied by the epochal theory of time. Each phase in the genetic process presupposes the entire quantum, and so does each feeling in each phase. The subjective unity dominating the process forbids the division of that extensive quantum which originates with the primary phase of the subjective aim. The problem dominating the concrescence is the actualization of the quantum in *solido*.

The above passages seem to have annoyed many commentators of *Process and Reality*. The genetic analysis of an actual occasion (Part III) divides the concrescence into primary, intermediate, and final phases, which, according to Whitehead, are not "in" the physical (i.e. coordinate) time. One phase of genetic divisions must be prior to another: but what sort of priority is this? William Christian discusses and *rejects* four possible ways of interpretation, i.e. (i) priority in physical time,(ii) the logical priority of a premise to a conclusion, (iii) a whole-part relation, and (iv) a dialectical process of the Hegelian development of an idea. Then he says, "though genetic priority may have some analogies with other sorts of priority, we must accept it as something of its own kind, but he does not analyse further the *sui generis* character of genetic divisions." [ii] Charles Hartshorne also questions the validity of "genetic" analysis, and proposes to accept only the succession of phases in the physical time.[iii]

What I will show in this paper is the importance of the distinction between "genetic" and "coordinate" analysis and its relevance to the interpretation of quantum physics, especially the relation of Heisenberg's indeterminacy principle to temporality, Bohr-Einstein debates, and the recent experimental refutation of the Bell Inequality .

If we take into consideration the impact of quantum physics on *the emergence of Whitehead's metaphysics*, as Lewis Ford shows in detail in his book[iv], we naturally expect that the "epochal" theory of time has something to do with the quantum "jump", or the discontinuous transition from potentiality to actuality. But we need some cautions. The references of quantum physics in *Science and the Modern World* (1925) is mainly to the primary stage of quantum theory in the early 1920's, and there is no textual evidence concerning whether Whitehead knows the final stage of quantum physics established by Bohr, Heisenberg, Schrördinger and other contemporary physicists. The composition of *Process and Reality* began at the Gifford Lectures in 1927, and the same year was memorable to the history of quantum physics: Bohr stated his principle of "complementariry" and stressed the "individuality" of quantum event in his Como Lectures, and Heisenberg published his paper of Indeterminacy Principle in *Zeitschrift für Physik*. Only two years later, *Process and Reality* was published (1929): although Whitehead did not mention Bohr's principle of "complementarity", nor Heisenberg's indeterminacy principle, there are indeed a striking correspondence between Whitehead's metaphysical analysis of an actual occasion on the one hand and Bohr's and Heisenberg's physical analysis of quantum events on the other hand.

The purpose of this paper is not to confirm or disconfirm the historical influence of Bohr's or Heisenberg's ideas on Whitehead's metaphysics. That is an interesting study in itself, but will remain only a conjecture. Rather, I will consider the problem of temporality in the interpretation of Heisenberg's indeterminacy principle, and then discuss Bohr's concept of "individuality" of quantum events under the Whiteheadian perspective. I will show that Whitehead's distinction between "genetic" and "coordinate" analysis of an actual occasion proves to be relevant to the interpretation of the delayed-choice experiment in quantum physics: this experiment is about the "indeterminate" past, which will catch the attention of process thinkers who take the "determinate" past for granted and think that only the future is indeterminate.

Lastly, I will present a new approach of quantum logic to analyse Bohr's concept of "individuality" of a quantum event. This approach uses the concept of "divisibility" of an event by another event, and defines the concept of "commensurability" of events. Then I will characterize the classical world by saying that all events are commensurable with each other whereas the quantum world is characterized by saying that some events are incommensurable with each other. This analysis may be interesting to Whiteheadian scholars because it will teach us that the concept of "individuality" of an quantum event **denies atomism** in so far as atomism presupposes the divisibility of an complex event into atomic component events. Many scholars think of Whitehead's epochal theory of time as "temporal atomism", and arbitrarily conjecture the existence of a temporal atom with a very minute scale of duration. Once we accept the quantum logical analysis and apply it to the epochal theory of time, we will understand the key concept is the "individuality" of an actual occasion and not "atomism" of any kind.

2. Heisenberg's Indeterminacy Principle and Time: Is there the indeterminate past?

In *the Physical Principles of the Quantum Theory* Heisenberg insists on the essential contingency of the future in his famous principle of indeterminacy. This principle states that we cannot exactly predict the future state of a physical system on the basis of the knowledge of the past. What we can do is only the probabilistic prediction which is testable, not by a single experiment, but only by statistical ensembles. The contingency of the future in this sense is a common notion of physicists today who have accepted the *Copenhagen* interpretation of quantum physics.

Suppose the future behavior of a photon coming in the half-silvered mirror (beam splitter): if it reflects from the mirror, it will travel the route α, and if it goes through the mirror, it will travel another route β. We do not know which route the photon will choose in advance, but can confirm afterward its choice by using the photo-detector: the exchange of energy between the photon and the photo-detector will testify the particle-character of a photon which cannot travel simultaneously two different routes. If the photo-detector E at the end of the route α clicks, then the photo-detector F at the end of the route β does not click, and *vice versa*. The classical description of a particle presupposes determinism *in principle* without any reference of an observer. The need of statistics only shows *our* ignorance of the physical system and there is nothing indeterminate in the system itself both in the past and in the future.

Against such a standpoint the Copenhagen interpretation of quantum physics proposes the indeterminacy principle. Statistical treatise is essential because we cannot predict the future behavior of a particle exactly in the individual case because of the indivisible relation of an observer and observed. Heisenberg refrains from talking about "reality" of the intermediate state of a physical system between our actual observations.

When applied to the future event, the indeterminacy principle *seems* reasonable to common-sense because common-sense well knows future contingency. But what about past contingency? If the indeterminacy principle stresses the indivisible connection between the observer and the observed, does the same principle apply not only to the future but also to the past? Not omniscient about the past, we have often to retro-dict, i.e. conjecture about what happened in the past, on the basis of present data. That the past is determinate at all its aspects without any observer is the postulate of physical realism, which can not be accepted unconditionally by the Copenhagen interpretation. Then must we accept the idea of the "indeterminate" past in some sense in quantum physics?

Indeed Heisenberg was aware of this question and discussed it in *The Physical Principles of the Quantum Theory* (1930):[v]

> The Indeterminacy Principle does not refer to the past: if the velocity of the electron is at first known and the position then exactly measured, the position for times previous to the measurement may be calculated. Then for these past times $\Delta p \Delta q$ is smaller than the usual limiting value, but this knowledge of the past is of a purely speculative character, since it can never (because of the unknown change in momentum caused by the position measurement) be used as an initial condition in any calculation of the future progress of the electron and thus cannot be subjected to experimental verification. It is a matter of a personal belief whether such a calculation concerning the past history of the electron can be ascribed any physical reality or not.

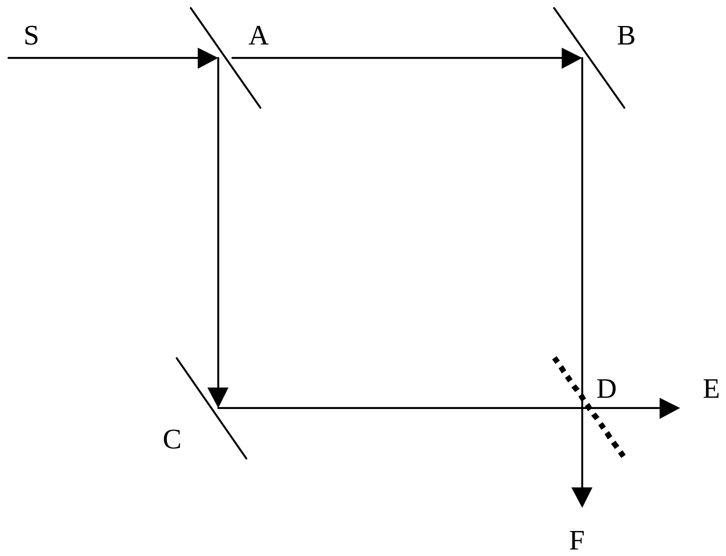
In the above citation Heisenberg did not reject the idea of the "indeterminate" past, but thought that such an idea was "purely speculative" character, and " matter of a personal belief" because it cannot be subjected to experimental verification. To Heisenberg at the 1920's only the prediction of the future was important, and the mathematical theory assisted him to calculate the probability of the end-state given the initial state: the description of the intermediate development of the system between two objectively recorded or recordable states did not seem to correspond to physical reality.

On the other hand, Einstein, as a critic of quantum physics, did not admit Heisenberg's standpoint, especially that the indeterminacy principle does not refer to the past. In the paper "Knowledge of Past and Future in Quantum Mechanics"(1931), Einstein proposed an imaginary experiment, in which "the possibility of describing the past path of one particle would lead to predictions as to the future behavior of a second particle of a kind not allowed in the quantum mechanics." [vi] So Einstein concluded that "the principle of the quantum mechanics must involve an indeterminacy in the description of past events which is analogous to the indeterminacy in the prediction of future events."

This should be understood in the context of Einstein's argument against the "completeness" of quantum physics just in the same way that the purpose of EPR argument (1935) was to show that the "completeness" of quantum physics would lead to absurdity. In other words, Einstein did not positively assert the existence of indeterminate past events, but only intended to deduce it as the necessary conclusion of the "completeness" of quantum physics.

The problem of the "indeterminate" past re-appeared about fifty years later in J. A. Wheeler's discussion of the "delayed-choice" experiment. This experiment is not an imaginary but an actual one which uses one particle (say, photon) instead of two particles in Einstein's case. [vii]

Fig. Schematic diagram of Wheeler's delayed choice experiment



Laser light incident on a half-silvered mirror A divides into two beams: one is along the path ABD(α), and the other is along the path ACD(β). In the above experimental arrangement a detection of a given photon by either by E or F suffices to determine which of the two alternative routes the photon will have traveled. This is the particle mode of the experiment. The photon travels either the route α or the route β.

If, now, a second half-silvered mirror D is inserted at the crossing points, the two beams are recombined , part along the route into E, and part along the route into F. This will cause wave interference effects, and the strengths of the beams going into E and F respectively will then depend on the relative phases of the two beams at the point of recombination. These phases can be altered by adjusting the path lengths, thereby essentially scanning the interference pattern. And this is the wave mode of the experiment. The photon travels in some way both routes, α and β, at the same time.

Now the crucial point is that the decision of whether or not to insert the second half-silvered mirror D can be left until a given photon has almost arrived at the cross-over point. Thus one decides whether the photon "shall have come by one route, or by both routes" after it has "*already done* its travel".

After confirming the fact that what we can say of past events is decided by (delayed)choices made in the near past and now, Wheeler discusses the possibility that the phenomena called into being by the present decision can reach backward in time, even to the earliest days of the universe. He says:[viii]

> To use other language, we are dealing with an elementary act of creation. It reaches into the present from billions of years in the past. It is wrong to think of the past as "already existing" in all detail. The "past" is theory. The past has no existence except as it is recorded in the present. By deciding what questions our quantum registering equipment shall put in the present we have an undeniable choice in what we have the right to say about the past.

The interpretation of the indeterminacy principle will be altered if we accept the concept of the past indeterminacy. Heisenberg originally considered this principle as the limit of the exactitude of two incommensurable quantities at the simultaneous measurement. But the indeterminacy of past events which have not been recorded, have a connection, not with their simultaneous measurability, but rather with the definability of their historic routes. That the definition of the past route or history of a particle depends on the present choice of an experimenter is the meaning of the "indeterminate past".

3. Bohr's Framework of Complementarity

Bohr's principle of "complementarity" is more closely connected with the "individuality" of a quantum event rather than with the indeterminacy principle. His emphasis is mainly on the definition of a quantum process, and not on the unavoidable "disturbance" or "physical influence" of the observer on the observed. His arguments rest on the insight that in quantum physics "we are presented with the inability of the classical frame of concepts to comprise the peculiar feature of indivisibility, or "individuality", characterizing the elementary process. The quantum paradox arises from "the apparent contradiction between the exigencies of the general superposition principle of the wave description and the feature of individuality of the elementary atomic processes." Stressing the impossibility of any sharp separation between the behavior of atomic objects and the interaction with the measuring instruments which serve to define the conditions under which the phenomena appear, he writes:[ix]

> The individuality of the typical quantum effects find its proper expression in the circumstance that any attempt of subdividing the phenomena will demand a change in the experimental arrangement introducing new possibilities of interaction between objects and measuring instruments which in principle cannot be controlled. Consequently, evidence obtained under different experimental conditions cannot be comprehended within a single picture, but must be regarded as complementary in the sense that only the totality of the phenomena exhausts the possible information about the objects.

Bohr distinguishes two modes of the description of a quantum process, which are "complementary but exclusive": one is "the space-time coordination", and the other is "the claim of causality". The two modes of description, though united in the classical theories, should be considered as "complementary but exclusive features of the description" in quantum physics.

Heisenberg summed up the framework of complementarity in the following diagram.[x]

<u>Space-time Coordination</u>	Alternatives related statistically	<u>Causal Relationship</u>
Quantum process is described in terms of space and time.		expressed by mathematical laws
But		But
Indeterminacy principle		Physical description of phenomena in space-time is impossible

Now we are ready to compare Bohr's (physical) framework of complementarity with Whitehead's (metaphysical) distinction between "coordinate" and "genetic" analysis of an actual occasion. The structurally similar arguments really characterize the theory of "concrescence" and space-time coordination in *Process and Reality*. Whitehead's discussion of causality, efficient or final, belongs to the *genetic analysis* of an actual occasion, i.e. in his *Theory of Prehension*. The internal development of "concrescence" of an actual occasion is the theme of this analysis. But according to Whitehead this internal process itself does not occur in physical time. Physical time makes its appearance in the "coordinate" analysis of the "satisfaction". Each phase in the genetic process presupposes the entire quantum---that is the point of the "epochal" theory of time.

The distinction between genetic and coordinate analyses has a bearing on the divisibility of the space-time region. The region of an actual occasion is divisible according to the coordinate analysis, but is undivided in the genetic growth. So the epochal theory of time stresses the coordinately indivisible character of an actual occasion.

Commentators of Whitehead's metaphysics, as far as I know, do not seem to have grasped fully the "individuality" of an actual occasion. The epochal theory of time was usually discussed through Zeno's paradox of motion and change, and considered as the metaphysical postulate which makes it possible for us to talk about "becoming" at all. I do not say that it is wrong, but only that such a metaphysical postulate is not sufficient to the concrete analysis of "becoming" and its relation to space-time.

For example, the epochal theory of time is often characterized as "temporal atomism".[xi]

But "atomism" is not a happy word in the sense that it has a connotation of mechanical world-view. Whitehead's standard usage is "the cell theory of actuality", and not "atomic theory of actuality". Moreover, an "individual" quantum event is not necessarily microscopic. The simultaneous correspondence of the EPR experiment shows us the "individuality" of a quantum process at a long (macroscopic) distance. The delayed-choice experiment shows us that the individual quantum process may have the "indeterminate past" according to the coordinate divisions of space-time. So the region of an individual quantum process may have an arbitrary size with respect to space-time coordinates.

In the next section I will analyse the concept of "individuality" by using a quantum logical analysis and show that the concept of completeness which Einstein presupposes in his criticism of quantum physics is irreverent to the quantum world.

4. A quantum logical Analysis of the Indivisibility, or "Individuality" of an Event

There is a hidden presupposition when we apply classical logic to the empirical world, namely, that all events are divisible with each other, or that all events are commensurable. This presupposition fails to be the case in the quantum world, so that one of the most fundamental laws of classical logic cannot claim universal validity.

In order to define the "divisibility" and "commensurability" of events, some preliminary discussions are in order.

Suppose two events, a and b:

a: The wind will blow tomorrow. b: Rain will fall tomorrow.

The two statements which predict the weather tomorrow are, strictly speaking, not propositions. Truth-values of propositions must be eternal in the sense that they must be fixed independent of the time when they are stated by any speaker. We cannot fix truth-values of the above statements today unless we are determinists. Tomorrow we can verify these predictions, but we are not titled to assign truth-values to these contingent statements today. The truth-table approach of classical logic is irrelevant to the contingent world of quantum physics.

I will give an introduction of quantum logic, which is wider than classical logic in its application in the sense that it can analyse contingent events in addition to determinate propositions.

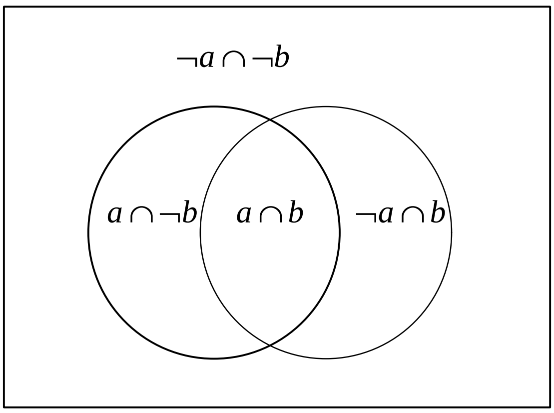
Now we define the divisibility of events as follows.

(1) aDb: The event a is divisible by the event b

$$aDb \Leftrightarrow_{def} a = (a \cap b) \cup (a \cap \neg b)$$

The right-hand side of this definition is the equivalence which we implicitly assume in our every day talk. "The wind will blow tomorrow" is equivalent to "The wind will blow and rain will fall tomorrow, or the wind will blow and rain will not fall tomorrow."

We can use Ven's diagram to visualize the divisibility of events.



Divisibility of events shown in Ven's diagram presupposes Boolean algebraic structure which characterizes classical logic.

It is noteworthy that both Wittgenstein's *Tractatus* and Russell's "*Logical Atomism*" implicitly presuppose that all events are divisible with each other: their analysis depend on the truth-table. Classical logic with the Boolean algebraic structure is the background of their world-view. If we denote $p_1 = a \cap b$ $p_2 = a \cap \neg b$ $p_3 = \neg a \cap b$ $p_4 = \neg a \cap \neg b$, we may call p_k $(1 \le k \le 4)$ atomic events because they make up the non-overlapping and exhaustive set of events. So we can decompose $a, \neg a, b, \neg b$ into atomic events p_k. In such a logical atomism, "the world divide into facts and the fact consists of atomic events (das Bestehen von Sachverhalten)----if we use Wittgenstein's phrase in *Tractatus*. The set of atomic events are called the logical spectrum of the world.

Note that the logical sprectrum of the world may be relative to our descriptive language. It may be simple and rough (as in the above case) or may be very fine(as in the case of a professional weather forecaster). Atomic events may be decomposed by another logical spectrum, and we will get the description of the world more and more in the detail. The point of logical atomism is not the existence of absolutely atomic events, but the divisibility of any event into more atomic events.

The usual formulation of quantum logic does not use the truth table, to say nothing of Ven's diagram. Quantum logical analysis seems difficult in its treatise. So I will give a very simple and understandable semantical definition of quantum logic by using the concept of divisibility.

We define the concept of "commensurability" of events :

(2) aCb : the event a is commensurable with the event b

$$aCb \Leftrightarrow_{def.} aDb \,\&\, bDa$$

Note that "&" is the symbol of semantics (meta-language), and must be distinguished from "$\cap$" of the object-language.

Now we can define the classical world in the following way.

(3) The world W is classical $\Leftrightarrow_{def} (\forall a \forall b)\{(a \in W \,\&\, b \in W) \rightarrow aCb\}$

In the classical world all events are commensurable with each other, i.e. all events are divisible with each other. Let denote by **0** the null-event which is stipulated by the contradictory statement such as $a \cap \neg a$, and by **1** the all-inclusive event which is stipulated by the tautology such as $a \cup \neg a$. Then we can give a natural meaning to the concept of the "complete" description of the classical world by using the logical spectrum defined as follows:

The set of non-overlapping and exhaustive events B={b_k} is called "logical spectrum",

i.e. $i \ne j, \rightarrow b_i \cap b_j = 0$ *and* $\bigcup_k b_k = 1$

In the classical world, any event *a* can be decomposed into the logical sum of atomic events with respect to the logical spectrum B as $a = \bigcup_k (a \cap b_k)$.

More generally, if there are multiple logical spectra $B^1, B^2, B^3, \ldots .. B^l$,

a can be decomposed as $a = \bigcup_{k(l)} (a \cap b^1_{k(1)} \cap b^2_{k(2)} \cap \ldots \cap b^l_{k(l)})$, where $b^l_{k(l)}$ is a member of B^l and $\bigcup_{k(l)}$ is the sum of all possible combinations of $b^l_{k(l)}$.

If we confine ourselves to the classical world which contains finite number of events, then we can easily stipulate the condition of the "complete description" of the classical world.

Let use the abbreviation $w_m = b^1_{k(1)} \cap b^2_{k(2)} \cap \ldots \cap b^l_{k(l)}$

Then we may say that the whole set of logical spectra $\{B^1, B^2, B^3, \ldots .. B^l\}$ gives the complete description of the classical world if $a \cap w_m = 0$ *or* $a \cap w_m = w_m$ for any event *a*, that is any event can be decomposed to the logical sum of w_m, and we need not any additional logical spectrum. Each w_m may be called an (absolutely) atomic event, and logical atomism is a suitable characterization of the classical world.

If we assign the equal probability for every atomic event w_m, then we can calculate the a priori probability of the event *a*.

$P(a) = \sum_{m(a)} P(w_{m(a)}) = \frac{n}{N}$ where N is the total number of atomic events, and n is the number of m(a) such that $a \cap w_m = w_m$.

The above description of the classical world is the logical basis of classical physics. The divisibility of any event $a = (a \cap b) \cup (a \cap \neg b)$ is always presupposed, and we may say that one of physicists' aims is to invent a new kind of logical spectrum so that we may get nearer to the ideal of the "complete" description of the world.

Quantum physics tells us that the divisibility formula $a = (a \cap b) \cup (a \cap \neg b)$ does not always hold. In the other words, there is a case in which the event a is indivisible by the event b. For example, let *a* be "the spin of the electron is up along x-axis", and *b* be "the spin if the same electron is up along y-axis", then a is indivisible by b, because of the indeterminacy principle.

Note that the indivisibility of an event *a* doesn't mean that a is indivisible by any other event, but that there are some events by which a is indivisible. So we can define the individuality of an event as follows:

(4) The event a has the character of "individuality" $\Leftrightarrow_{def.} (\exists x)(\sim aDx)$

As there exist quantum events with the character of "individuality", we cannot use the probabilistic formula $p(a) = p(a \cap b) + p(a \cap \neg b)$ when a is indivisible by b. As explained later, this is the reason why the Bell-Inequality breaks down in quantum events.

The next task is to define the quantum world through the concept of divisibility.

The quantum world is the world of quantum logic, which has the algebraic structure of orthomodular lattice. It is known that the orthocomplemented lattice is orthomodular if and only if the divisibility relation aDb is symmetrical.[xii] So we can define the quantum world as follows:

(5) The world W is a quantum world if the relation of divisibility is symmetric, and there exist incommensurable events in W.

W is a quantum world

$$\Leftrightarrow_{def.} \{(\forall a,b)(a \in W \,\&\, b \in W) \rightarrow (aDb \rightarrow bDa)\} \,\&\, (\exists a,b)(a \in W \,\&\, b \in W \,\&\, \sim aCb)$$

The distinction between "classical" and "quantum" worlds is analogous to the situation in relativity physics in which Riemann geometry as a generalization of Euclidean geometry holds. Some theorems of Euclid geometry as applied empirical data are not necessarily true in the relativistic world especially when gravitational effects are strong. Just in the similar way, some laws of the classical logic as applied to empirical data are not necessarily true in the quantum world when we observe incommensurable events.

As we can get classical logic by adding the condition of commensurability to quantum logic, quantum logic should not be considered as a “weird” or “strange” logic invented by logicians. Rather, quantum logic is more faithful, than classical logic, to the concrete situation of experimental contexts because it does not presuppose the dogmatic thesis of divisibility of all events. Classical logic is embedded in the deterministic world of classical physics where all propositions have the prefixed truth-values independent of observers. The atomistic view of events is implicitly presupposed in classical physics. On the other hand, quantum physics, where irreducible contingency appears in the context of observation, admits the existence of an “indivisible” event. The individuality of a quantum event can involve the macroscopic spatio-temporal extension. It can extends over two temporally distant locations which involve “the indeterminate past” in the coordinate division as in the delayed-choice experiment.

In the next section we discuss the indivisibility of a quantum event which extends over two spatially distant locations, i.e. the so-called EPR correlation.

5 Bell's Inequality as an analytical result in the classical world

The experimental test of Bell's Inequality which the French physicist Alain Aspect conducted in 1982 attracted the attention of those who were interested in philosophical problems of quantum physics. This experiment manifested one of the most paradoxical characteristics of quantum system, namely the non-separability of two contingent events, concerning the correlation of polarized photon pairs at a distance. Both philosophers and physicists were reminded of the celebrated debate between Bohr and Einstein about the completeness of quantum mechanics in the 1930s. The imaginary experiment, which Einstein used in his polemics against the alleged completeness of quantum mechanics, became a real one through the progress of technology. The combination of conceptual analysis and experimental tests revived the controversy about the philosophical status of quantum physics in the new light. The test of Bell's theorem became a starting point for refreshed research into the nature of quantum events.

In this section I will deduce the generalized Bell Inequality as an analytical (necessary) result in the classical world which I have defined in the previous section. The classical assumption about the "divisibility" of an event into the atomic components causes Bell's inequality. We need only an elementary theory of probability and information, and not any additional physical knowledge to derive Bell's inequality. [xiii]

Suppose that $A=\{a_i\},\ B=\{b_j\}$ are two logical spectra, e.g., the set of observable values in physical experiments. According to the information theory, if we measure A and B, and get the value a_i *and* b_j, the newly acquired information is

$$I(a_i) = -\log p(a_i) \qquad I(b_j) = -\log p(b_j)$$

The joint information of a_i *and* b_j is

$$I(a_i \cap b_j) = -\log p(a_i \cap b_j)$$

Similarly, the conditional information of a_i *given* b_j is

$$I(a_i \mid b_j) = -\log p(a_i \mid b_j)$$

According to Bayes's Theorem,

$$I(a_i \cap b_j) = I(a_i \mid b_j) + I(b_j) = I(b_j \mid a_i) + I(a_i)$$

The mean value of information of A and B is, respectively,

$$H(A) = \sum_i p(a_i) I(a_i) \qquad H(B) = \sum_j p(b_j) I(b_j)$$

Let the expectation of the joint information of (A and B) be **H (A∩B)**

$$H(A \cap B) = \sum_{i,j} p(a_i \cap b_j) I(a_i \cap b_j)$$

The conditional information of A given b_j is

$$H(A|b_j) = \sum_i H(a_i|b_j) = \sum_i p(a_i|b_j) I(a_i|b_j)$$

The conditional information of A given B is the mean value of $H(A|b_j)$

$$H(A|B) = \sum_j p(b_j) H(A|b_j) = \sum_{i,j} p(a_i \cap b_j) I(a_i|b_j)$$

The new formulation of Bayes's theorem is

$$H(A \cap B) = H(A|B) + H(B) = H(B|A) + H(A)$$

Let $I(a_i;b_j)$ be the correlation information between a_i *and* b_j

$$I(a_i;b_j) = I(a_i) - I(a_i|b_j) = I(b_j) - I(b_j|a_i)$$

Though the value of this correlation information may be positive or negative, its mean value must be non-negative according to Gibb's theorem.[xiv]

$$H(A;B) = \sum_{i,j} p(a_i \cap b_j) I(a_i;b_j) = \sum_j p(b_j)(\sum_i p(a_i|b_j) \log \frac{p(a_i|b_j)}{p(a_i)}) \geq 0$$

So we can get the fundamental inequality

(6) $H(A|B) \leq H(A) \leq H(A \cap B)$

Now we can deduce the generalized Bell Inequality from (6)

Suppose α and β are two separated physical systems. α has two logical spectra **A** and **A**'. β has also two logical spectra B and B'. Let $a_i, a'_k\, b_j, b'_l$ be discrete observable values of A, A', B, and B'.

In the classical world where all events are divisible with each other, there exists the joint probability for every possible combination such as $p(a_i \cap a'_k \cap b_j \cap b'_l)$.

So we can generalize the fundamental inequality (6) as follows:

(7) $H(A \cap B') \leq H(A \cap B \cap A' \cap B') = H(A|B \cap A' \cap B') + H(B|A' \cap B') + H(A'|B) + H(B')$

Using $H(A|B \cap A' \cap B') \leq H(A|B),\qquad H(B|A' \cap B') \leq H(B|A')$

we subtract $H(B')$ from both sides of (7),and get the generalized Bell Inequality as follows:

(8) Bell-I $H(A|B') \leq H(A|B) + H(B|A') + H(A'|B')$

We can rewrite (8) in the symmetrical form by using the concept of information distance introduced by Schumacher.[xv]

Defining the information distance $\mathrm{d}(A,B)$ between A and B as

$$\mathrm{d}(A,B) = H(A|B) + H(B|A)$$

we get the symmetrical representation of the generalized Bell inequality.

(9) Bell-2: $\mathrm{d}(A,B') \leq \mathrm{d}(A,B) + \mathrm{d}(B,A') + \mathrm{d}(A',B')$

Note that the inequality (7) presupposes "commensurability" between A and A' and between B and B'. So this inequality lost its meaning in the case of incommensurable observables.

On the other hand, the inequality (8) is meaningful in the quantum world becase only commensurable pairs of observables (A and B) ,(A and B'), (A',and B), and (A'and B') appear. So we can empirically test the Bell inequality (8) to decide whether our world is classical or not. The result of empirical test by Aspect and others clearly shows that the Bell Inequality does not hold. This experiment is analoguous to the "crucial" experiments of genneral theory of relativity which tell us that our world is non-Euclidian, i.e. not "flat" space-time. There is an exprimentarily testified sense in which we say that our world is not a classical world.

Then what about the "completeness" of quantum physics against which Einstein protested?

Does the disconfirmation of the Bell Inequality prove the "completeness" of quantum physics? As I agrued elsewhere[xvi], the valid conclusion of EPR arguments and Bell's theorem, even if we accept classical presuppositions, is the non-locality of an indivisible quantum event and not the "incompleteness" of quantum physics. Einstein's concept of "completeness" of a physical theory implicitly presupposes the classical world where the relation of divisibility holds. In the quantum world where incommensurable (not mutually divisible) events exist, the very concept of "completeness" does not hold. Therefore, we must say that quantum physics is neither incomplete nor complete in the classical sense.

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