A Model-Predictive-Control-Based Torque Demand Control Approach for Parallel Hybrid Powertrains

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Abstract—In this paper, a torque-demand-based control approach is developed for parallel hybrid powertrains that consist of a torque distributor, a load observer, and two feedback control loops for an internal combustion engine and an electric motor, respectively. The torque distributor is composed of the torque demand, torque split, torque compensation, and torque limit. The torque control law for the engine is constructed with model predictive control based on a nonlinear mean-value model. A proportional–integral (PI) observer is designed to estimate the torque load of the powertrain, which is Lyapunov stable. For the electric motor, a linear model predictive control law is designed with current feedback. To validate the proposed torque demand control approach, simulation results that were conducted on a simulator are demonstrated, in which full-scaled dynamics of the powertrain are simulated.

Index Terms—Hybrid powertrain, model predictive control, proportional–integral (PI) observer, torque control, torque demand.

NOMENCLATURE

- $T_{Le}$: Torque load of the IC engine.
- $T_{air}$: Intake air temperature.
- $T_{acs}$: Torque demand of vehicular accessories.
- $T_{bat}$: Torque demand of battery recharging.
- $T_{pmax}$: Maximum torque of electric path.
- $T_{pmin}$: Minimum torque of electric path.
- $T_{brk}$: Torque demand of the braking pedal.
- $T_{cpt}$: Torque compensation by the electric motor.
- $T_{dE}$: Torque demand of the IC engine.
- $T_{dM}$: Torque demand of the electric motor.
- $T_{dem}$: General torque demand of the powertrain.
- $T_E$: Actual output torque of the IC engine.
- $T_{los}$: Nominal output torque of the IC engine.
- $T_{loss}$: Torque loss of the IC engine.
- $T_{on}$: Output torque of the electric motor.
- $T_{pe}$: Torque value for the current speed of the IC engine in the peak-efficiency curve.
- $T_{req}$: Torque requirement of the IC engine.
- $T_{req}$: Torque request of the electric motor.
- $T_{rem}$: Torque requirement of the electric motor.
- $T_{rc}$: Torque demand of the acceleration pedal.
- $U_{max}$: Maximum value of the voltage vector of the electric motor.
- $V_d$: Displacement of the IC engine.
- $V_m$: Intake manifold volume of the IC engine.
- $i_d$: $d$-axis current of the electric motor.
- $i_q$: $q$-axis current of the electric motor.
- $i_{dmax}$: Maximum value of the $d$-axis current of the motor.
- $i_{dmin}$: Minimum value of the $d$-axis current of the motor.
- $i_{qmax}$: Maximum value of the $q$-axis current of the motor.
- $i_{qmin}$: Minimum value of the $q$-axis current of the motor.
- $k_p$, $k_i$: Proportional and integral gains, respectively.
- $k_{pm}$: Proportional gain of the proportional–derivative (PD) compensator.
- $k_{dm}$: Differential gain of the PD compensator.
- $p$, $m$: Number of predictive steps for system states and control inputs, respectively.
- $p_m$: Intake manifold pressure of the IC engine.
- $p_{max}$: Maximum intake manifold pressure of the engine.
- $p_{min}$: Minimum intake manifold pressure of the engine.
- $n_p$: Number of pole pairs of the electric motor.
- $r_p$, $w_i$, $u_d$, $u_q$: $d$- and $q$-axis voltage of the motor, respectively.

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In this paper, a formulated approach of torque-based engine control strategies for a downsized SI engine from simulation design to final validation on a 145 demonstration car [5]. The previous literature focuses on the description of modular functions and implementary means for the torque-based control of the conventional powertrain by 148 using IC engines only. In this paper, a formulated approach of torque-based control is designed to control a hybrid powertrain 150 that consists of an IC engine and an electric motor.

In terms of the control for hybrid powertrains, Salmasi reviews control strategies for hybrid electric vehicles and 153 discusses the pros and cons of each approach in detail [6]. Sundstrom et al. propose a rule-based method of computing the torque split factor to choose the sizes of the IC engine and electric motor [7]. Adhikari et al. propose an online power-flow-control strategy for fully hybridized parallel hybrid electric vehicle based on the power-balancing strategy so that the 159 IC engine is operated at its peak-efficiency region by using the 160 electrical system [8]. Based on the aforementioned description, 161 it can be known that the torque-based control approach is not 162 yet applied to the control of hybrid powertrains. In this paper, a 163 model predictive torque control (MPC) approach is designed to 164 directly manipulate the torque of a hybrid powertrain.

In the case of the application of MPC, Borhan and Vahidi design an online optimization-based predictive controller to 167 control the power-split hybrid electric vehicle [9]. Rotenberg developed an MPC strategy for power management, which 169 achieves better fuel economy than the rule-based approach [10]. Suzuki et al. used an MPC controller to implement the indi-vidual air–fuel estimation and control [11]. Bageshwar et al. used the MPC algorithm to compute the spacing–control laws for 173 transitional maneuvers of vehicles equipped with adaptive cruise control [12]. Based on the aforementioned review, the 175 MPC algorithm has been applied to many branches of auto-motive research. In this paper, a nonlinear MPC algorithm is employed to control a hybrid powertrain. In fact, Saerens et al. illustrate the capabilities of MPC for the control of conventional 179 powertrains instead of hybrid powertrains [13]. Saerens et al. 180 take the driving resistance that was computed by the vehicle speed as the load of the powertrain, which results in poor 182 dynamic performance of the controller. In this paper, a PI observer is designed to estimate the transient torque load of the 184 hybrid powertrain to improve the dynamic performance of the 185 MPC controller.

In this paper, an MPC-based torque demand control approach is developed to implement the torque-based control of a parallel hybrid powertrain. This approach is composed of a torque dis-tributor, a torque-based nonlinear MPC controller of an engine, a torque-based linear MPC controller of an electric motor, and a torque load observer. The torque distributor includes a torque compensation of the powertrain by using the PD compensator. In the end, this approach is validated by some simulation scenarios in MATLAB/Simulink.

This paper is organized as follows. The torque demand con-trol approach is illustrated in Section II. Two MPC controllers...
II. TORQUE DEMAND CONTROL

The architecture of the parallel hybrid powertrain that is considered here is sketched in Fig. 1. As shown this figure, the hybrid powertrain consisted of the torque demand controller, IC engine, permanent-magnetic synchronizer motor (PMSM), clutch, transmission, differential, wheels, and some other accessories. It is obvious that the hybrid powertrain has the following two power plants connected in parallel: 1) the IC engine and 2) the PMSM. At the same time, both the engine and PMSM can operate together or individually to drive the wheels.

A. Description of the Control Approach

As illustrated in Fig. 2, the torque demand control approach of the parallel hybrid powertrain is composed of the torque distributor, nonlinear MPC controller of the engine, linear MPC controller of the PMSM, and load observer. Based on the acceleration pedal position $\phi_a$, the braking pedal position $\phi_b$, the engine speed $\omega_e$, the torque requirement $T_{acs}$ of vehicle accessories, and the torque requirement $T_{bat}$ of battery recharging, the torque distributor computes the general torque demand $T_{\text{dem}}$ of the powertrain. Based on the powertrain states and a suitable proportional factor $\lambda$, the general torque demand $T_{\text{dem}}$ is further divided into the following two parts: 1) engine torque demand $T_{dE}$ and 2) motor torque demand $T_{dM}$.

Then, the linear MPC controller receives the motor torque demand $T_{dM}$ and other PMSM signals to calculate the current command of the $q$-axis $i_q$ to manipulate the PMSM. At the same time, the nonlinear MPC controller uses the engine torque demand $T_{dE}$ and engine torque load $T_{Le}$ from the load observer to compute the throttle angle command $\theta_c$ to manipulate the engine. In fact, the powertrain load is immeasurable. In this paper, a PI observer is designed to estimate the torque load of the parallel hybrid powertrain.

Note that, based on the aforementioned description of the torque demand control, the control approach of the parallel hybrid powertrain can be divided into two levels: The high level generates the torque demand of the IC engine and electric motor, i.e., $T_{dE}$ and $T_{dM}$, whereas the low level computes the value of control signals for the actuation of power sources based on the torque demand.

B. Torque Distributor

As shown in Fig. 3, the torque distributor can be divided into the following four parts:

- torque demand;
- torque split;
- torque compensation;
- torque limit.

The function of torque demand is to generate the general torque demand $T_{\text{dem}}$ of the powertrain based on the requirement of drivers. Then, the general torque demand $T_{\text{dem}}$ is divided by the torque split into the following two parts based on a suitable proportional factor $\lambda$: 1) the engine torque requirement $T_{e}$ and 2) the motor torque requirement $T_{m}$.

With regard to some other methods, checking the constraints on the battery state of charge (SOC) and limits of rate of torque/speed change of the IC engine and PMSM is generally prior to the torque split. However, as illustrated in Fig. 3, this paper first calculates the split ratio and then corrects it according to the operational limits of the components. The reason is that safety is first. The torque limit is assigned to the last step, which can ensure that the value of the torque control command is always within the safe range limited by the current condition.

In this paper, we assume that the torque generation dynamics of the PMSM are linear. That is, the motor current is completely proportional to the output torque of the motor. However, the
Fig. 4. Torque supply map of the parallel hybrid powertrain. L1 is of the maximum output torque of the hybrid powertrain. L2 is of the maximum output torque of the hybrid powertrain through the peak-efficiency curve of the IC engine. L3 is of the maximum output torque of the IC engine. L4 is of the peak-efficiency curve of the engine. L5 is of the maximum output torque of the PMSM. L6 is of the minimum output torque of the PMSM. L7 is of the iso-torque curves of the IC engine. L8 is of the iso-throttle position curves. L9 is of the driving resistance curve.

torque generation dynamics of the engine is nonlinear in reality; therefore, it is impossible for the engine torque demand \( T_{dem} \) to be completely realized by the MPC controller. Hence, a PD compensator is designed to compute a torque compensation \( T_{cpt} \) to compensate for the torque error, which is the difference between the engine torque demand \( T_{dem} \) and the actual output \( T_E \). Finally, both the motor torque requirement \( T_{rM} \) and the torque compensation \( T_{cpt} \) are added together as the motor torque request \( T_{rM} \).

In reality, the output torque of the electric motor is limited by the SOC of the battery. In the torque limit block, both the maximum torque \( T_{brm} \) and the minimal torque \( T_{lmin} \) of the electric path are used to revise the torque request \( T_{rM} \) of the electric motor. In the end, both the engine torque demand \( T_{dem} \) and motor torque demand \( T_{dM} \) are achieved and exported to the MPC controller.

1) Torque Demand: The general torque demand \( T_{dem} \) of the hybrid powertrain is computed from the acceleration pedal \( \phi_a \), torque demand \( T_{acs} \) of the engine, and brake supply \( T_{brk} \). Based on the engine map illustrated in Fig. 4, the pedal torque demand \( T_{dem} \) and motor torque demand \( T_{dM} \) are achieved and exported to the MPC controller.

2) Torque Split: One of the key issues is how we can split the general torque demand \( T_{dem} \) into the engine torque requirement \( T_{re} \) and the electric motor torque requirement \( T_{rM} \). As illustrated in Fig. 4, there is a torque supply map of the parallel hybrid powertrain that consists of an IC engine and an electric motor. In this figure, it is easy to see that the torque supply range of the parallel hybrid powertrain is between L6 and L1. To enhance fuel economy, however, it is better to make the IC engine operate at the L4 peak-efficiency curve. Hence, the objective of torque split is to make the engine operate around 305 L4 under the whole range of engine speed. Then, the torque supply range of the powertrain includes the following two parts: 1) between L5 and L6 and 2) between L2 and L4.

To make the IC engine operate at L4 as closely as possible, the engine throttle command \( \phi_e \) is not in accordance with the acceleration pedal position \( \phi_a \) during the transient operation of the IC engine. For example, from points A to D in Fig. 4, there are two tracks: A–B–D and A–C–D. Usually, when the 313 accelerated pedal changes, the throttle position follows, i.e., the engine operating at track A–C–D. To make the IC engine operate at the peak-efficiency curve L4, track A–B–D is better 316 to follow. However, a torque difference between the points B and C occurs, and the electric motor is then employed to 318 generate a torque \( T_{rM} \) to compensate it. In this paper, the motor 319 torque requirement \( T_{rM} \) is determined through a torque split 320 factor \( \lambda \in [-1, 1] \) as

\[
T_{rM} = \lambda T_{dem}
\]

where the value of \( \lambda \) can be calculated by the following two 322 conditions: 1) braking and 2) driving.

**Braking**: During the braking, i.e., \( T_{brk} > 0 \), the split factor is expressed as follows:

\[
\lambda = \begin{cases} 
0, & \text{ABS action} \\
1, & \text{Others.}
\end{cases}
\]

Based on (3), it can be known that the regeneration braking 326 will stop when the antilock brake system (ABS) is active, i.e., 327 wheels locking. For other conditions, e.g., \( T_{brk} > T_{bmax} \), the 328 lacking braking torque is compensated by the hydraulic brake 329 system.

**Driving**: During the driving, \( \lambda \) can also be computed 331 by the following two conditions: 1) recharging and 2) assist. 332 Recharging means that the SOC of the battery is lower than a 333 threshold and the battery needs to be recharging. Assist denotes 334 that the SOC is larger than the threshold and the battery can 335 output the electricity.

\[
\lambda = \begin{cases} 
0, & \text{Kickoff for engine} \\
1, & \text{Kickoff for motor} \\
\frac{T_{dem} - T_{brk}}{T_{dem}}, & \text{otherwise}
\end{cases}
\]

where kickoff means that the switch at the end of acceleration 339 pedal is active.

In (4), the first item means that, when the kickoff switch is 341 active, the hybrid powertrain enters into the sport mode. The 342 second item means that, when the vehicle speed is low, the 343 hybrid powertrain enters into the pure-electric-driving mode. The 344 last item means that the powertrain enters into the hybrid- 345 driving mode.
Recharging: When the battery is recharging by using the electric motor, i.e., $T_{bat} > 0$, the split factor $\lambda$ is expressed as follows:

$$
\lambda = \begin{cases} 
0, & \text{Kickoff} \\
-\frac{T_{dem} - T_{pe}}{T_{dem}}, & T_{dem} \geq T_{bat} + T_{pe} \\
-\frac{T_{brk} + T_{dem} - T_{pe}}{T_{dem}}, & T_{pe} < T_{dem} < T_{bat} + T_{pe} \\
-\frac{T_{brk} + T_{dem}}{T_{dem}}, & T_{dem} \leq T_{pe}.
\end{cases}
$$

(5)

In (5), the first item denotes that the electric motor does not regenerate when the kickoff switch is active. The second, third, and fourth items are together to make the engine operate around the peak-efficiency curve. The last item denotes that, when $T_{dem} = 0$, the hybrid powertrain enters into the recharging mode.

In fact, $\lambda$ has eight states of operation. The switched way of the torque split factor $\lambda$ among the eight states is described in Appendix A.

Then, the engine torque requirement $T_{re}$ is given by

$$
T_{re} = \begin{cases} 
0, & T_{brk} > 0 \\
0, & T_{bat} = 0 \& v_a < v_{th} \\
\dot{\lambda}, & \text{Kickoff} \\
T_{pe}, & \text{Others}.
\end{cases}
$$

(6)

In (6), the first item describes that, during the brake, the IC engine stops. The second item describes that, when the vehicle speed is lower than the threshold $v_{th}$, the hybrid powertrain enters into the pure-electric-driving mode. The third item describes that, when the kickoff switch is active, the engine outputs the maximum power. The last item describes that the engine may operate around the peak-efficiency curve for other conditions.

Note that, in (6), it is shown that the target of the torque split is to make the engine operate around the peak-efficiency curve through the electric motor. The excess torque that is produced by the ICE engine will be regenerated by the electric motor during the driving. The lacking torque between $T_{dem}$ and $T_{pe}$ is compensated by the electric motor.

The general torque demand is divided into the following two parts: 1) the engine torque requirement $T_{re}$ and 2) the motor torque requirement $T_{rm}$.

3) Torque Compensation: In fact, due to some nonlinear characteristics of engine dynamics, it is usually difficult for the engine torque demand to completely be realized by the MPC controller. Because the $g$-axis current of the motor is proportional to its output torque, we can suppose that the torque demand of the PMSM can completely be realized. Therefore, the torque error $\dot{\epsilon}$ between the engine torque demand $T_{dE}$ and the actual torque $\dot{T}_E$ can be compensated by the PMSM.

Here, a PD compensator is employed to compute the value of torque compensation $T_{cpt}$ as follows:

$$
T_{ cpt } = k_{ pm } \dot{\epsilon} + k_{ dm } \frac{d \dot{\epsilon}}{dt},
$$

(7)

In (7), the first formula utilizes the PD principle to compute the torque compensation $T_{cpt}$. In the second formula in (7), the estimated output torque of the IC engine $\dot{T}_E$ is calculated by a mean-value model, which is formulated in the next section.

Then, the motor torque request $T_{RM}$ is given as follows:

$$
T_{RM} = T_{RM \_ max} + T_{cpt}.
$$

(8)

In (8), the lack of output torque that was derived from the nonlinearity of the IC engine dynamics is added into the torque request $T_{RM}$ of the electric motor.

4) Torque Limit: The output torque of the electric path, including the electric motor and battery, is limited by their own states, e.g., the SOC of the battery. Therefore, the torque request $T_{RM}$ of the PMSM can be limited as follows:

$$
T_{dM} = \begin{cases} 
\min \{T_{RM}, T_{bmax}\}, & T_{RM} \geq 0 \\
\max \{T_{RM}, T_{bmin}\}, & T_{RM} < 0.
\end{cases}
$$

(9)

In (9), the first formula shows a motor torque limit of the driving mode, and the second shows a motor torque limit of the braking mode.

Then, the engine torque demand $T_{dE}$ can be revised as follows:

$$
T_{dE} = \begin{cases} 
T_{re} + T_{RM} - T_{dM}, & T_{brk} = 0 \\
0, & T_{brk} > 0.
\end{cases}
$$

(10)

In (10), the first item denotes that the IC engine compensates for the lack of torque electric path for the driving mode, i.e., $T_{brk} = 0$. However, the engine torque demand $T_{dE}$ does not change for the braking mode, i.e., $T_{brk} > 0$.

Through the torque distributor, both the engine torque demand $T_{dE}$ and the motor torque demand $T_{dM}$ are achieved and then transferred to the MPC controller of the engine and motor.

III. DESIGN OF CONTROLLER

As aforementioned, the torque demand control approach includes the following two MPC controllers: 1) the nonlinear MPC controller of the IC engine and 2) the linear MPC controller of the PMSM. A PI observer is also employed to estimate the load torque of the hybrid powertrain.

A. Nonlinear MPC Controller of the Engine

Due to the nonlinearity of engine dynamics, a nonlinear MPC controller will be designed by the mean-value model in this section.

1) Dynamic Model of the Engine: The mean-value model can be employed to compute the output torque of the IC engine by the intake manifold pressure $p_m$. The intake manifold dynamics, crankshaft dynamics, and combustion torque output $T_{ccht}$ are, respectively, given as follows [14]

$$
\left\{ \begin{array}{l} 
\dot{\epsilon}_e = \frac{1}{\rho} (T_{ccht} - T_{los} - T_{e}) \\
\dot{p}_m = \frac{\eta_i \delta \eta_{col} Q_{m} R_{air} T_{air}}{\rho_{V_m} V_m} u_{th} \\
T_{ccht} = \frac{\eta \delta \eta_{col} C_{T} V_{d}}{4 \pi R_{air} T_{air}} p_m.
\end{array} \right.
$$

(11)

(12)

Note that, in the aforementioned dynamic model of the 426 engine, the derivative of temperature is neglected, because the 427
intake manifold temperature is assumed constant. In general, 429 the temperature derivative can be neglected, because it has only 430 a minor effect on the manifold pressure dynamics.

2) State-Space Model: In this paper, it is supposed that the 431 engine loss torque $T_L$ is within a small range of engine speed 432 and is similarly proportional to the engine speed $w_e$, based on a 433 suitable factor $\eta_e$, i.e., $T_L = \eta_e w_e$. As depicted in (11), we 434 can choose the state variables as $x_1 = \omega_e$, $x_2 = p_m$, and the control input as $u = u_{th}$. Then, the engine model can be rewritten as 437 the following state-space equation:

$$
\begin{align}
\dot{x}_1 &= -a_1 x_1 + a_2 x_2 - d \\
\dot{x}_2 &= -a_3 x_1 x_2 + bu \\
y[k] &= cx_2[k]
\end{align}
$$

whereas the parameters are given by $a_1 = \eta_e / I_d$, $a_2 = 438 (\eta_h \eta_f C_T V_d) / (4\pi I_d R_{air} T_{air})$, $a_3 = \eta_f V_d / 4\pi V_m$, $d = 439 T_L / I_d$, and $b = (\eta_h \theta T_{air} / R_{air} V_d) / V_m$.

3) Discrete-Time Model: Assume that the torque demand 442 for the engine is delivered in discrete-time fashion with interval 443 $T_s$, i.e., the desired reference signal for torque generation 444 given at time $t = kT_s$ is $y_d[k + 1|k], y_d[k + 2|k], \ldots$, where 445 $k$ denotes the sampling index with sampling period $T_s$. To 446 design a desired control law in the discrete-time framework, 447 we first make a one-step prediction of the system (13) through 448 a control input increment $\Delta u[k]$ at time $k$, which is equivalent 449 to discretizing (13) into the following form:

$$
\begin{align}
\dot{x}_1[k + 1|k] &= a_7 x_1[k] + a_8 x_2[k] - d_0 \\
\dot{x}_2[k + 1|k] &= -a_9 x_1[k] x_2[k] + x_2[k] + h_1 (u[k - 1] + \Delta u[k]) \\
y[k] &= cx_2[k]
\end{align}
$$

where the system output $y[k]$ is the combustion torque $T_{cd}$ 451 defined in (12), and the other parameters are given by $a_7 = 452 1 - a_1 T_s$, $a_8 = a_2 T_s$, $a_9 = a_3 T_s$, $d_0 = d T_s$, $h_1 = b_0 T_s$, 453 and $c = (\eta_f \eta_h C_T V_d) / 4\pi R_T$.

In (14), the control input $u[k]$ at time $k$ is computed by backward difference, which is equivalent to an embedded integrator 456 for the control input as follows:

$$
u[k] = u[k - 1] + \Delta u[k]
$$

4) Nonlinear MPC Controller: Iteratively using this idea for 459 the two-step-ahead prediction, under the condition that the control input keeps the value $u[k + 1|k] = u[k]$, i.e., $\Delta u[k + 1|k] = 0$, 460 yields

$$
\begin{align}
\dot{x}_1[k + 2|k] &= -a_8 a_9 x_1[k] x_2[k] + a_9^2 x_1[k] + p_1 x_2[k] - p_2 + a_3 b_0 u_2[k] \\
\dot{x}_2[k + 2|k] &= -a_9 a_2 x_1[k] x_2[k] + p_2 x_2[k] + a_7 a_3^2 x_1[k] x_2[k] + a_9 a_3^2 x_1[k] x_2[k] + h_2 (u[k - 1] + \Delta u[k]) \\
y[k] &= cx_2[k]
\end{align}
$$

where $p_1 = a_8 a_7 + 1$, $p_2 = a_7 (d_0 + 1)$, $p_3 = a_9 (a_7 + a_9 d_0 + 1)$, 463 $p_4 = a_9 d_0 + 1$, and $h_2 = -a_7 a_9 b_0 x_1[k] - a_9 a_9 b_2 x_2[k] + a_9 a_9 b_0 d_0 + 2 b_0$.

The design problem of the control system considered can 466 then be presented as follows. For given torque demand subse- 467 quence $y_d[k + 1|k], y_d[k + 2|k], \ldots$, find a feedback control law for even increment $\Delta u[k]$ of every predictive step. Solve the 486 following moving horizontal optimization problem [15]:

$$
\begin{align}
\min_{\Delta u[k]} \sum_{i=1}^{p} w_i (\dot{y}[k + i|k] - y_d[k + i|k])^2 + \sum_{j=1}^{m} r_j \Delta u[k]^2 \\
\text{subject to } (14)-(17)\text{ and }
\end{align}
$$

where $\dot{y}[k + i|k]$ are the predictions for the system output based 471 on the predictive states $\hat{x}[k + i|k]$.

Now, we adopt a two-step prediction of system states and 473 a one-step prediction of control input, i.e., $m = 1, p = 2$. Substituting (14), (17) and (16) into (18), the cost function can be rewritten as follows:

$$
J(\Delta u[k]) = (r w_1 \epsilon_1^2 h_1^2 + w_2 \epsilon_2^2 h_2^2) \Delta u^2[k] - 2 c (w_1 \epsilon_1 [k + 1|k] + w_2 \epsilon_2 [k + 2|k]) \Delta u[k] + \epsilon^2[k + 1|k] + \epsilon^2[k + 2|k] \\
$$

where

$$
\begin{align}
\epsilon[k + 1|k] &= y_d[k + 1|k] - c (-a_9 x_1[k] x_2[k] + x_2[k] + h_1 u[k - 1]) \\
\epsilon[k + 2|k] &= y_d[k + 2|k] - c (-a_9 x_1[k] x_2[k] + p_4 x_2[k] + a_7 a_9 x_1^2[k] x_2[k] + a_9 a_9 x_1[k] x_2[k] + h_2 u[k - 1]).
\end{align}
$$

As denoted in (21), it is easy to see that the problem is to 478 find the least value of the quadratic equation. Then, a straight forward calculation of the derivative to (21) yields the unique 480 optimal solution of cost function (18) as follows:

$$
\Delta u[k] = \frac{w_1 c h_1 \epsilon[k + 1|k] + w_2 c h_2 \epsilon[k + 2|k]}{r_1 + w_1 c^2 h_1^2 + w_2 c^2 h_2^2}. \\
$$

Thus, based on the mean-value model, a torque-biased two-step nonlinear MPC algorithm as depicted in (22) will be found. Based on the aforementioned deduction, it is shown that the 484 nonlinear MPC law has some characteristics as follows. First, as 485 shown in (13), the mean-value model used is nonlinear. Second, 486 as depicted in (22), the optimal solution of cost function is an 487 analytical expression, i.e., the control law is analytical. Finally, the 488 optimal solution, as denoted in (22), is unique.

B. Linear MPC Controller of the PMSM

A linear MPC controller is employed to predict the current 491 and torque behavior through a mathematical model of the 492 PMSM. The electrical dynamics of PMSM is given as follows: 493

$$
\begin{align}
\frac{d}{dt} i_d &= \frac{1}{L_m} (u_d - R_{im} i_d + \omega_m L_m i_q) \\
\frac{d}{dt} i_q &= \frac{1}{L_m} (u_q - R_{im} i_q - \omega_m L_m i_d - \omega_m \Psi_m). \\
\end{align}
$$
Assume that the rotor speed is constant in the predictive period, which indicates that \( \omega_{me} \) is a constant. Then, the voltage vector is selected as the control input \( U_u = [u_d, u_q]^T \), and the current vector is selected as state variables \( X_i = [i_d, i_q]^T \) on the \( d \)- and \( q \)-axes. With the aforementioned choice, the discrete-time model with sampling time \( T_s \) at the time \( k \) is given as follows:

\[
X_i[k+1|k] = AX_i[k|k] + BU_u[k|k] + E \tag{24}
\]

where \( A = T_s \begin{bmatrix} -\frac{R}{L} + 1 & \omega_{me} \\ -\omega_{me} & -\frac{R}{L} + 1 \end{bmatrix}, B = T_s/L \begin{bmatrix} 1 \\ 1 \end{bmatrix}, E = \begin{bmatrix} 0 \\ \omega_{me} \Psi_m \end{bmatrix} \), and \( U_u[k|k] \) is used as the control input at time \( k \), i.e.,

\[
U_u[k|k] = U_u[k-1] + \Delta U_u[k|k] \tag{25}
\]

where \( U_u[k-1] \), the control input of the previous step, is known at time \( k \).

Based on the predictive current of the \( d \)- and \( q \)-axes in (24), the torque of the PMSM in the future is predicted as follows:

\[
T_m[k|k] = C_m i_q[k|k]. \tag{26}
\]

Furthermore, let the torque demand of the electric motor be \( T_{dm} \). Because the number of predictive steps is equivalent to \( n \) for both the control input \( U_u \) and the system output \( T_m \), the cost function of the electric motor control is given as follows:

\[
\Delta U_u[k|k] = \min \left\{ \Delta U_u[k|k-1|k] \right\} \sum_{j=1}^{n} \lambda_{mj} \left( T_m[k+j|k] - T_{dm}[k+j|k] \right)^2 + \sum_{j=1}^{n} \rho_{mj} \Delta U_u^2[k|k] \tag{27}
\]

subject to (24)–(26), and

\[
\begin{align*}
|\Delta U_u[k|k]| & \leq U_{umax} \\
|U_u[k|k]| & \leq U_{umax} \\
i_{dmin} & \leq X_1[k|k] \leq i_{dmax} \\
i_{qmin} & \leq X_2[k|k] \leq i_{qmax}.
\end{align*} \tag{28}
\]

At time \( k \), substituting (24) and (26) into the cost function (27), it is straightforward to achieve the optimal control voltage increment \( \Delta U_u \) of the PMSM \( d \)- and \( q \)-axes for the optimal output to inverter as in (30), shown at the bottom of the page, where \( z = 1 \ldots n \).

As depicted in (30), the solution is a subsequence. However, the first term \( \Delta U_u[k|k] \) is used as the control input increment, and then, achieving the optimal control input, \( U_u[k|k] \) is applied to control the PMSM at time \( k \). This process is repeated every sampling time.

IV. DESIGN OF THE TORQUE LOAD OBSERVER

In practice, the torque load of the hybrid powertrain is an immeasurable variable in engineering. In the following context, a PI observer is designed to estimate the torque load of the IC engine.

Let the engine output torque \( T_e = T_{cht} - T_{los} \). Then, the crankshaft dynamics of (11) can be rewritten as follows:

\[
I \ddot{\omega}_c = T_e - T_{Le}. \tag{31}
\]

Assumption 1: At time \( k - 1 \), the load torque is \( T_{L0} \). Then, at time \( k \), the actual load \( T_{Le} \) can be described as follows:

\[
T_{Le} = T_{L0} + \Delta(\dot{\omega}_e) \tag{32}
\]

where \( \Delta(\dot{\omega}_e) \) is an unknown function that is bounded by a positive constant \( \zeta(\dot{\omega}_e) \), i.e., \( |\Delta(\dot{\omega}_e)| \leq \zeta(\dot{\omega}_e) \).

Based on the principle of the PI observer in Fig. 5, the load observer can be formulated as follows:

\[
\begin{align*}
\dot{T}_{Le} & = k_p \dot{\omega}_e + k_i \int \omega_e dt + \zeta(\dot{\omega}_e) \sgn(\dot{\omega}_e) \\
\dot{\omega}_e & = \frac{1}{I_d}(T_e - T_{Le}).
\end{align*} \tag{33}
\]

where \( \sgn(\cdot) \) is a signum function.

Note that, when the observer is applied to engineering practice by digital implementation, the estimated error may not tend to zero due to the limited sampling rate and quantization.

Let \( \sigma = k_i \int \omega_e dt \). Combining (31)–(33), the error equation can be given as follows:

\[
\begin{align*}
I_d \dot{\omega}_e & = \Delta(\dot{\omega}_e) + T_{L0} - k_p \dot{\omega}_e - \sigma - \zeta(\dot{\omega}_e) \sgn(\dot{\omega}_e) \\
\dot{\sigma} & = k_i \omega_e.
\end{align*} \tag{34}
\]

Theorem 1: Suppose that Assumption 1 holds. Then, for any positive constant \( k_p \) and \( K_i \), the error system (34) is Lyapunov stable.

Proof: Let \( \dot{\sigma} = T_{L0} - \sigma \). The error dynamics in (34) are rewritten as

\[
\begin{align*}
I_d \dot{\omega}_e & = \Delta(\dot{\omega}_e) - k_p \dot{\omega}_e - \sigma - \zeta(\dot{\omega}_e) \sgn(\dot{\omega}_e) \\
\dot{\sigma} & = -k_i \omega_e.
\end{align*} \tag{35}
\]
Consider the candidate of Lyapunov function as
\[
V = \frac{1}{2} \left( I_d \dot{\omega}_e^2 + \frac{1}{k_i} \sigma^2 \right).
\] (36)

Then
\[
\dot{V} = (\Delta(\dot{\omega}_e) - \xi(\dot{\omega}_e) \text{sgn}(\dot{\omega}_e)) \ddot{\omega}_e - k_p \ddot{\omega}_e^2.
\] (37)

Note that \(|\Delta(\dot{\omega}_e)| \leq \xi(\dot{\omega}_e)\); therefore
\[
\dot{V} \leq 0.
\] (38)

Equation (38) denotes that the error system is of Lyapunov stability. Furthermore, combining (35)–(38), it can be concluded that
\[
\dot{V} = 0 \iff \ddot{\omega}_e = 0, \quad \ddot{\theta}_L = -\Delta \iff \ddot{\omega}_e = 0, \quad T_{L0} + \Delta = \sigma. \quad (39)
\]

Based on the principle of LaSalle’s invariant set, \(\ddot{\omega}_e\) and \(\ddot{\sigma}\) converge to the set where \((\ddot{\omega}_e, \ddot{\sigma}) = (0, -\Delta)\) as \(t \to \infty\). Hence, combining (32) and (33) in the set \(\ddot{\omega}_e = 0\) and \(T_{L0} + \Delta = \sigma\), the estimated load torque is given as follows:
\[
\dot{T}_{Le} = \sigma = T_{L0} + \Delta.
\] (40)

Equation (40) implies that, in the invariant set, the estimated load torque \(\dot{T}_{Le} \to T_{Le}\) as \(t \to \infty\).

Remark 1: The aforementioned analysis does not consider the chattering from the switching of the signum function. A modified signum function \(\text{sgn}_m(\dot{\omega}_e)\) can attenuate the chattering in the practical application, which is given as follows:
\[
\text{sgn}_m(\dot{\omega}_e) = \begin{cases} 
\dot{\omega}_e, & |\dot{\omega}_e| \leq B_{th} \\
\text{sgn}(\dot{\omega}_e), & |\dot{\omega}_e| > B_{th}.
\end{cases}
\] (41)

With regard to the proportional and integral gains \(k_p, K_i\) of the torque load observer, the different powertrain needs different values. Li and his cooperators describe the tuning rules of PI gains in detail in [16].

V. SIMULATION VALIDATION

To verify the MPC approach of the parallel hybrid powertrain, some simulation scenarios are carried out in MATLAB/Simulink.

A. Description of the Simulation

The simulation platform of the parallel hybrid powertrain is constructed through MATLAB/Simulink. The parameters of the simulation model, including the engine, PMSM, and vehicle, are shown in the Appendix B.

In the following cases of simulation, the MPC algorithm of engine uses a two-step prediction of state variables, a one-step prediction of control input, and a one-step desired torque. The parameters of the engine model used in this simulation are detailed as follows: \(a_1 = 0.1251, a_2 = 0.0675, a_3 = 0.532, b = 1.968 \times 10^6, c = 0.0027, \) and \(T_s = 0.0001\). The weights are \(r = 7 \times 10^6\) and \(w_1 = w_2 = 1\). Likewise, the MPC algorithm of the electric motor also uses a two-step prediction of state variables, a one-step prediction of control input, and a one-step desired torque. The weights of the cost function for the PMSM are \(\rho_m = 0.72, \lambda_m = 1\).

B. Simulation Results

Two operating scenarios that consist of the step throttle and 590 load are tested on the simulation platform that was developed 591 for the parallel hybrid powertrain. The scenario of the step 592 throttle simulates a driver giving a step torque demand during 593 the normal operation to accelerate the vehicle. The scenario of 594 the step load simulates a driver abruptly turning on a power 595 accessory, e.g., air conditioner.

1) Case 1—Step Throttle With Hybrid and Conventional Powertrains: The simulation scenario of the step throttle is 598 employed to simulate the step increment of the acceleration 599 pedal from the driver. The aim of this scenario is to validate 600 the torque demand controller with the PD compensation and 601 PI observer. In the following context, two compared simulation 602 results are illustrated and analyzed. Both results employ the 603 torque demand controller, but one result aims at the hybrid 604 powertrain, whereas the other result aims at the conventional 605 powertrain.

Fig. 6 shows some torque signals of the step throttle 607 simulation. Fig. 6(a) shows some simulation results of the 608 engine torque for the hybrid powertrain. In this figure, the pre- 609 dictive torque from the MPC controller can follow the engine 610 torque demand \(T_{LE}\). However, it is obvious that there is an error 611 between the engine torque demand and the actual engine torque 612 due to the error of the mean-value model of the IC engine.

After 19 s, as shown in Fig. 6, a step increment of the torque 614 demand of about 50 Nm is applied to the powertrain. It is shown 615 that there is a response time delay between the predictive engine 616 torque and the actual torque. For the time delay, the motor- 617 assisted hybrid powertrain can provide an equivalent torque 618 compensation, as illustrated in Fig. 6(c), by using the electric 619...
motor. In this figure, it is also shown that the conventional powertrain does not supply torque compensation during the step throttle.

From 19.1 s to 19.4 s in Fig. 6(c), it is shown that the PMSM can output a torque to compensate for the hybrid powertrain at the initial phase of the step throttle. Compared with Fig. 6(a) and (c), it is shown that, after finishing the shift from gear 2 to gear 3, the MPC controller with the PD compensator can provide a torque compensation to make up for the time delay of the torque response of the IC engine.

Fig. 7 shows some simulation results of the general torque. For the hybrid powertrain, the general torque is the sum of the engine and motor torque, i.e., \( T_{E} + T_{M} \). For the conventional powertrain, the general torque is only the engine torque \( T_{E} \). In this figure, it is shown that the electric motor can regenerate the redundant torque of the IC engine between the torque demand and the actual torque from 19 s to 21 s. However, it is impossible for the conventional powertrain to do it. Hence, the IC engine can operate around the peak-efficiency curve by using the electric motor.

Fig. 8 shows some simulation results of the torque engine throttle, speed, and \( q \)-axis current of the electric motor. In Fig. 8(a), it is shown that the shifting of the hybrid powertrain lags behind that of the conventional because the MPC controller with the PD compensator regenerates the excess torque to the engine torque demand \( T_{dE} \). From 19.5 s to 21.2 s in Fig. 8, due to the PD compensation, the PMSM absorbs the excess torque of the powertrain, which results in the lag of shifting for the hybrid powertrain.

After engagement of the synchronizer from 22 s to 22.5 s, the engine speed of the hybrid powertrain is larger than the conventional due to the behavior of the compensated torque. Comparing Figs. 6(c) and 8(c), it is shown that the motor torque is proportional to the motor current at the \( q \)-axis. In Fig. 8(a), it is easy to figure out that the consumption of fuel is almost the same between the hybrid powertrain and the conventional.

Fig. 9 shows some simulation results of the clutch, gear position, and vehicle speed. In the simulation, the vehicle speed is calculated by the longitudinal-dynamics formula, in which the driving resistances include the braking force, air resistance, slope resistance, and roll resistance. In Fig. 9(a) and (b), a shift from gear 2 to gear 3 occurs from 21 s to 22 s for the hybrid powertrain and the conventional, respectively. In Fig. 9(c), the resulting vehicle speed still exceeds that of the conventional powertrain due to the torque compensation after the synchronization.

Based on Fig. 9(c), at 19.3 and 22.3 s, the torque compensation from the electric motor is beneficial to the acceleration of the vehicle, which makes the vehicle speed of the hybrid powertrain exceed that of the conventional. Due to the energy recovery from 19.5 s to 21.2 s, the PMSM may not cost additional electric energy.
In summary, based on the aforementioned analysis, it can be concluded that the torque demand controller with the PD compensator fits the hybrid powertrain.

2) Case 2—Step Load With Hybrid and Conventional Powertrains: The simulation scenario of the step load is employed to simulate a driver suddenly turning on a vehicle accessory, e.g., air conditioner, during normal driving. The aim of this scenario is to validate that the torque demand control strategy may diminish the effect of the step torque load. In the following context, two compared simulation results from both the hybrid powertrain and the conventional are illustrated and analyzed.

Fig. 10 shows some simulation results of the demand, actual, and predictive engine torque and the additional torque load of the powertrain. The predictive engine torque is from the MPC controller. Likewise, the predictive engine torque corresponds with the engine torque demand. Due to the error of the mean-value model, there is a difference between the predictive engine torque and the actual.

For the step load scenario, the engine torque demand is 150 Nm during the whole operation. At 10.7 s, a step torque of 50 Nm loaded on the powertrain. Compared with Fig. 10(a) and (b), it is shown that the shifting time of the hybrid powertrain advanced that of the conventional by about 1 s due to the torque compensation from the motor, as depicted in Fig. 10(c), from 10.7 s to 12.5 s. After synchronization of clutch, the PMSM outputs a torque to compensate for the slower torque response of the IC engine. When the torque demand controller has made sure that the negative effect from step torque load was diminished, the torque compensation from the PMSM is released.

Fig. 11 shows some simulation results of the general torque. Likewise, the general torque is the sum of the engine and motor torque \( T_E + T_M \) for the hybrid powertrain. For the conventional powertrain, the general torque is only the engine torque \( T_E \). From 10.7 s to 12.5 s, as shown in Fig. 11, the general torque demands significantly differ, irrespective of the same throttle commands and vehicle speed traces because the hybrid powertrain can utilize the electric motor to compensate for a torque of the step load, i.e., the step load from vehicle accessories can rapidly be compensated. However, the conventional powertrain cannot compensate for it, except for drivers changing the acceleration pedal position \( \phi_a \).

Fig. 12 shows some simulation results of the engine throttle, speed, and motor current at the \( q \)-axis. Fig. 12(a) shows that the consumption of fuel is also almost the same between the hybrid and the conventional powertrain. In Fig. 12(b), it is shown that the engine speed of the hybrid powertrain is larger than the conventional. Comparing Figs. 12(c) and 10(c), likewise, it is shown that the motor torque is proportional to the motor current at the \( q \)-axis.

Fig. 13 shows some simulation results of the clutch, gear position, and vehicle speed. In Fig. 13(a), the shifting time of the conventional powertrain lags behind that of the hybrid for about 1 s. In Fig. 13(a), the shift is from gear 2 to gear 3. At 10.7 s, a step torque of 50 Nm loaded on the powertrain.
11 s, as shown in Fig. 13(c), due to the torque compensation of the PMSM, the acceleration of the vehicle for the hybrid powertrain almost does not change, whereas the acceleration of the conventional powertrain obviously decreases. Therefore, it can be concluded that the actuation of the power accessory can lead to nonsmooth vehicle speed.

In summary, based on the aforementioned analysis, the torque demand controller can improve drivability, particularly a sudden demand torque loaded on the powertrain. Likewise, it can be concluded that the torque demand controller with the PD compensator fits the hybrid powertrain.

VI. CONCLUSION

An MPC-based torque demand control approach has been developed to implement the torque-based control of a parallel hybrid powertrain that consists of a torque distributor, a nonlinear MPC controller, a linear MPC controller of the PMSM, and a torque load observer.

Compared with other approaches, the proposed MPC control law of the IC engine is a nonlinear, analytic, and optimal solution to cost function, and therefore, it can be applied to engineering practice with very little computational effort only.

A torque distributor with the PD compensator is designed to make the IC engine operate around the peak-efficiency curve. The PD compensator can compensate for the torque error between the engine torque demand and the actual engine torque during normal operation.

A PI observer is designed for the torque demand controller to estimate the torque load of the hybrid powertrain, which is Lyapunov stable. Compared with the conventional method, calculating the driving resistance by vehicle speed, the PI observer has a better dynamic performance.

### APPENDIX A

**TORQUE SPLIT FACTOR**

<table>
<thead>
<tr>
<th>Mean of the Torque Split Factor</th>
<th>Type of operation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda = -1$</td>
<td>Positive</td>
</tr>
<tr>
<td>$\lambda \in (-1, 0)$</td>
<td>Partial recharging using engine</td>
</tr>
<tr>
<td>$\lambda = 0$</td>
<td>Provide all torque using engine</td>
</tr>
<tr>
<td>$\lambda \in (0, 1)$</td>
<td>Purely conventional braking</td>
</tr>
<tr>
<td>$\lambda = 1$</td>
<td>Provide torque by engine and motor</td>
</tr>
<tr>
<td></td>
<td>Partial regenerative braking</td>
</tr>
</tbody>
</table>

In this paper, it has been assumed that the torque generation of the PMSM is proportional to the current of the $q$-axis. In fact, it is nonlinear. Therefore, the PD compensator of the torque demand controller should be employed to compensate for the torque error of the general torque demand instead of the IC engine torque error. In addition, the torque demand control approach will be validated in the test bench of a parallel hybrid powertrain and even on a vehicle in the future.

### APPENDIX B

**PARAMETERS OF THE SIMULATION PLATFORM FOR THE PARALLEL HYBRID POWERTRAIN**

The parameters used for the simulation platform for the parallel hybrid powertrain are listed as follows.

<table>
<thead>
<tr>
<th>Type</th>
<th>V6.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Injection</td>
<td>Port injection.</td>
</tr>
<tr>
<td>Displacement</td>
<td>2.9 L.</td>
</tr>
<tr>
<td>Maximum Power</td>
<td>189 kW/(6400 r/min).</td>
</tr>
<tr>
<td>Maximum Torque</td>
<td>300 Nm/(4800 r/min).</td>
</tr>
<tr>
<td>Stator inductance</td>
<td>0.00015 H.</td>
</tr>
<tr>
<td>Stator resistance</td>
<td>0.01 $\Omega$.</td>
</tr>
<tr>
<td>Flux induced by the magnet</td>
<td>0.12 Wb.</td>
</tr>
<tr>
<td>Pole pairs</td>
<td>4.</td>
</tr>
<tr>
<td>Motor inertia</td>
<td>0.4 kg/m².</td>
</tr>
<tr>
<td>Vehicle cross section</td>
<td>2 m².</td>
</tr>
<tr>
<td>$C_w$-value of the vehicle</td>
<td>0.3.</td>
</tr>
<tr>
<td>Vehicle mass</td>
<td>1250 kg.</td>
</tr>
<tr>
<td>Tire–road friction coefficient</td>
<td>0.01.</td>
</tr>
<tr>
<td>Dynamic tire radius</td>
<td>0.35 m.</td>
</tr>
<tr>
<td>Differential transmission ratio</td>
<td>3.7.</td>
</tr>
<tr>
<td>Transmission</td>
<td>6 Gear.</td>
</tr>
<tr>
<td>Reverse-gear ratio</td>
<td>3.5.</td>
</tr>
</tbody>
</table>

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REFERENCES


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AUTHOR PLEASE ANSWER ALL QUERIES

AQ1 = Part of the Acknowledgment was captured in the affiliation note. Please check if this is correct.
AQ2 = Email addresses of authors found in their bios were captured in the affiliation note. Please check.
AQ3 = PD was defined as proportional-derivative. Please check if this is correct. Otherwise, provide the corresponding definition.
AQ4 = MPC was defined as model predictive controller. Please check if this is correct. Otherwise, provide the corresponding definition.
AQ5 = Please check if the editing made did not change the sentence’s intended meaning.
AQ6 = Acronyms that were mentioned only once should be spelled out, hence the omission of EMS. Please check if this is appropriate.
AQ7 = Notes I-IV were rephrased as Note that.... Please check if this is correct. Otherwise, please advise if each note should be captured as a section heading.
AQ8 = Please check if the editing made did not change the sentence’s intended meaning.
AQ9 = Please check if the editing made did not change the sentence’s intended meaning.
AQ10 = Please check if insertion of the article the did not change the sentence’s intended meaning.
AQ11 = Please check if the editing made did not change the sentence’s intended meaning.
AQ12 = Please check if the editing made did not change the sentence’s intended meaning.
AQ13 = Please check if wording of the lead-in sentence for the definition list is correct.
AQ14 = Please check if Psi should be captured as Ψ.
AQ15 = Friction coefficient tyre/street was changed to tire-road friction coefficient. Please check if this is correct.
AQ16 = Please check if wording of the Acknowledgment is correct.
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Abstract—In this paper, a torque-demand-based control approach is developed for parallel hybrid powertrains that consist of a torque distributor, a load observer, and two feedback control loops for an internal combustion engine and an electric motor, respectively. The torque distributor is composed of the torque demand, torque split, torque compensation, and torque limit. The torque control law for the engine is constructed with model predictive control based on a nonlinear mean-value model. A proportional–integral (PI) observer is designed to estimate the torque load of the powertrain, which is Lyapunov stable. For the electric motor, a linear model predictive control law is designed with current feedback. To validate the proposed torque demand control approach, simulation results that were conducted on a simulator are demonstrated, in which full-scaled dynamics of the powertrain are simulated.

Index Terms—Hybrid powertrain, model predictive control, proportional–integral (PI) observer, torque control, torque demand.

NOMENCLATURE

A, B, C Coefficient matrices of the electric motor dynamics model. If there is no special declaration, all quantities adopt the International System of Units (SI) in the context.

$B_{th}$ Predefined threshold for the signum function used by the proportional–integral (PI) observer.

$C_m$ Electric motor torque constant $C_m = \frac{3}{2} p_n \Phi_m$.

$C_T$ Specific constant of the internal combustion (IC) engine.

$I_d$ Equivalent moment of inertia of the powertrain.

$L_m, R_{em}$ Motor inductance and resistance, respectively.

$q_{th}$ Maximum flow rate of throttle.

$Q_{th}$ Gas constant of air.

$T_{Le}$ Torque load of the IC engine.

$T_{air}$ Intake air temperature.

$T_{acs}$ Torque demand of vehicular accessories.

$T_{bat}$ Torque demand of battery recharging.

$T_{E}$ Maximum torque of electric path.

$T_{min}$ Minimum torque of electric path.

$T_{brk}$ Torque demand of the braking pedal.

$T_{cbt}$ Combustion torque of the IC engine.

$T_{cpt}$ Torque compensation by the electric motor.

$T_{dem}$ Torque demand of the powertrain.

$T_{dem}$ General torque demand of the powertrain.

$T_{E}$ Actural output torque of the IC engine.

$T_{los}$ Nominal output torque of the IC engine.

$T_{los}$ Torque loss of the IC engine.

$T_{m}$ Output torque of the electric motor.

$T_{pe}$ Torque value for the current speed of the IC engine.

$V_d, V_m, i_d, i_q, i_{q_{min}}, i_{q_{max}}, i_{q_{min}}, i_{q_{max}}$ Maximum value of the voltage vector of the electric motor.

$V_d$ Displacement of the IC engine.

$V_m, i_d, i_q, i_{q_{min}}, i_{q_{max}}, i_{q_{min}}, i_{q_{max}}$ Torque requirement of the electric motor.

$V_m$ Maximum torque of electric path.

$V_{pe}$ Torque demand of the braking pedal.

$W_{loss}$ Torque loss of the IC engine.

$W_{loss}$ Minimum value of the $d$-axis current of the motor.

$W_{loss}$ Maximum value of the $q$-axis current of the motor.

$W_{loss}$ Current command of the $q$-axis of the motor.

$W_{loss}$ Maximum value of the $q$-axis current of the motor.

$W_{loss}$ Minimum value of the $q$-axis current of the motor.

$W_{loss}$ Proportional and integral gains, respectively.

$W_{loss}$ Proportional gain of the proportional–derivative (PD) compensator.

$W_{loss}$ Differential gain of the PD compensator.

$W_{loss}$ Number of predictive steps for system states and control inputs, respectively.

$W_{loss}$ Intake manifold pressure of the IC engine.

$W_{loss}$ Maximum intake manifold pressure of the engine.

$W_{loss}$ Minimum intake manifold pressure of the engine.

$W_{loss}$ Number of pole pairs of the electric motor.

$W_{loss}$ Weighting coefficients of the controller.

$W_{loss}$ $d$- and $q$-axis voltage of the electric motor.
I. INTRODUCTION

OVER the last few decades, a great number of researches have been devoted to find new technical solutions that may improve the fuel economy and emission performances of vehicles. However, under the current technical conditions, the hybrid powertrain is regarded as a promising system that can reduce energy consumption and pollutant emission through different power plants such as the IC engine and electric motor. Usually, some efficiency maps are produced by steady-state testing [1]. Then, the maps are used to control the hybrid powertrain to satisfy drivers’ requirements of vehicle speed, which may be called speed-based control in this paper. However, some dynamic effects are not included in the maps, that is, it is impossible to implement the transient torque control of the powertrain by the approach. It is known that the torque-based control approach can transiently manipulate both the torque and speed of the powertrain. Therefore, the torque-based control approach will be employed to control the hybrid powertrain in this paper.

In fact, there are some researches on the torque-based control of powertrain in the literature. Gerhardt et al. first introduced a new engine management system by torque-based control in 1998 [2]. Heintz et al. describe a torque-based engine control architecture that uses a central torque demand variable to control the regulating qualities [3]. Gerhardt et al. focus on the introduction of function software module, and Heintz et al. show an algorithm of functional module. Yutaro et al. estimate the intake air mass from the correlation between intake valve lift characteristics and air quantity that enter the engine and utilize the potential of a variable valve actuation system for engine torque control [4]. Sollecie et al. present a developing way of torque-based engine control strategies for a downsized SI engine from simulation design to final validation on a 145 demonstration car [5]. The previous literature focuses on the description of modular functions and implementary means for the torque-based control of the conventional powertrain by148 using IC engines only. In this paper, a formulated approach of torque-based control is designed to control a hybrid powertrain that consists of an IC engine and an electric motor.

In terms of the control for hybrid powertrains, Salmasi reviews control strategies for hybrid electric vehicles and discusses the pros and cons of each approach in detail [6]. Sundstrom et al. propose a rule-based method of computing the torque split factor to choose the sizes of the IC engine and electric motor [7]. Adhikari et al. propose an online power-157 flow-control strategy for fully hybridized parallel hybrid electric vehicle based on the power-balancing strategy so that the 159 IC engine is operated at its peak-efficiency region by using the 165 electrical system [8]. Based on the aforementioned description, 166 it can be known that the torque-based control approach is not yet applied to the control of hybrid powertrains. In this paper, a model predictive torque control (MPC) approach is designed to directly manipulate the torque of a hybrid powertrain.

In the case of the application of MPC, Borhan and Vahidi design an online optimization-based predictive controller to control the power-split hybrid electric vehicle [9]. Rotenberg et al. developed an MPC strategy for power management, which achieves better fuel economy than the rule-based approach [10]. Suzuki et al. used an MPC controller to implement the individual air–fuel estimation and control [11]. Bageshwar et al. used the MPC algorithm to compute the spooling–control laws for transitional maneuvers of vehicles equipped with adaptive cruise control [12]. Based on the aforementioned review, the MPC algorithm has been applied to many branches of auto-motive research. In this paper, a nonlinear MPC algorithm is employed to control a hybrid powertrain. In fact, Saerens et al. illustrate the capabilities of MPC for the control of conventional powertrains instead of hybrid powertrains [13]. Saerens et al. take the driving resistance that was computed by the vehicle speed as the load of the powertrain, which results in poor dynamic performance of the controller. In this paper, a PI observer is designed to estimate the transient torque load of the hybrid powertrain to improve the dynamic performance of the MPC controller.

In this paper, an MPC-based torque demand control approach is developed to implement the torque-based control of a parallel hybrid powertrain. This approach is composed of a torque distributor, a torque-based nonlinear MPC controller of an engine, a torque-based linear MPC controller of an electric motor, and a torque load observer. The torque distributor includes a torque compensation of the powertrain by using the PD compensator. In the end, this approach is validated by some simulation scenarios in MATLAB/Simulink.

This paper is organized as follows. The torque demand control approach is illustrated in Section II. Two MPC controllers
The architecture of the parallel hybrid powertrain is sketched in Fig. 1. As shown in this figure, the hybrid powertrain consisted of the torque demand controller, IC engine, permanent-magnetic synchronizer motor (PMSM), clutch, transmission, differential, wheels, and some other accessories. It is obvious that the hybrid powertrain has the following two power plants connected in parallel: 1) the IC engine and 2) the PMSM. At the same time, both the engine and PMSM can operate together or individually to drive the wheels.

### II. TORQUE DEMAND CONTROL

The architecture of the parallel hybrid powertrain that is considered here is sketched in Fig. 1. As shown in this figure, the hybrid powertrain consisted of the torque demand controller, IC engine, permanent-magnetic synchronizer motor (PMSM), clutch, transmission, differential, wheels, and some other accessories. It is obvious that the hybrid powertrain has the following two power plants connected in parallel: 1) the IC engine and 2) the PMSM. At the same time, both the engine and PMSM can operate together or individually to drive the wheels.

#### A. Description of the Control Approach

As illustrated in Fig. 2, the torque demand control approach of the parallel hybrid powertrain is composed of the torque distributor, nonlinear MPC controller of the engine, linear MPC controller of the PMSM, and load observer. Based on the acceleration pedal position \( \phi_a \), the braking pedal position \( \phi_b \), the engine speed \( \omega_e \), the torque requirement \( T_{\text{acs}} \) of vehicle accessories, and the torque requirement \( T_{\text{bat}} \) of battery recharging, the torque distributor computes the general torque demand \( T_{\text{dem}} \) of the powertrain. Based on the powertrain states and a suitable proportional factor \( \lambda \), the general torque demand \( T_{\text{dem}} \) is further divided into the following two parts: 1) engine torque demand \( T_{\text{dE}} \) and 2) motor torque demand \( T_{\text{dM}} \).

Then, the linear MPC controller receives the motor torque demand \( T_{\text{dM}} \) and other PMSM signals to calculate the current demand of the \( q \)-axis \( i_{qc} \) to manipulate the PMSM. At the same time, the nonlinear MPC controller uses the engine torque demand \( T_{\text{dE}} \) and engine torque load \( T_{LE} \) from the load observer to compute the throttle angle command \( \theta_c \) to manipulate the engine. In fact, the powertrain load is immeasurable. In this paper, a PI observer is designed to estimate the torque load of the parallel hybrid powertrain.

Note that, based on the aforementioned description of the torque demand control, the control approach of the parallel hybrid powertrain can be divided into two levels: The high level generates the torque demand of the IC engine and electric motor, i.e., \( T_{\text{dE}} \) and \( T_{\text{dM}} \), whereas the low level computes the value of control signals for the actuation of power sources based on the torque demand.

#### B. Torque Distributor

As shown in Fig. 3, the torque distributor can be divided into the following four parts:

- torque demand
- torque split
- torque compensation
- torque limit

The function of torque demand is to generate the general torque demand \( T_{\text{dem}} \) of the powertrain based on the requirement of drivers. Then, the general torque demand \( T_{\text{dem}} \) is divided by the torque split into the following two parts based on a suitable proportional factor \( \lambda \): 1) the engine torque requirement \( T_{\text{re}} \) and 2) the motor torque requirement \( T_{\text{rm}} \).

With regard to some other methods, checking the constraints on the battery state of charge (SOC) and limits of rate of torque/speed change of the IC engine and PMSM is generally prior to the torque split. However, as illustrated in Fig. 3, this paper first calculates the split ratio and then corrects it according to the operational limits of the components. The reason is that safety is first. The torque limit is assigned to the last step, which can ensure that the value of the torque control command is always within the safe range limited by the current condition.

In this paper, we assume that the torque generation dynamics of the PMSM are linear. That is, the motor current is completely proportional to the output torque of the motor. However, the
In this figure, it is easy to see that the torque supply 300 range of the parallel hybrid powertrain is between L6 and L1. 302 To enhance fuel economy, however, it is better to make the 303 IC engine operate at the L4 peak-efficiency curve. Hence, the 304 objective of torque split is to make the engine operate around 305 L4 under the whole range of engine speed. Then, the torque 306 supply range of the powertrain includes the following two parts: 307 1) between L5 and L6 and 2) between L2 and L4. 308

To make the IC engine operate at L4 as closely as possible, 309 the engine throttle command $\phi_e$ is not in accordance with the 310 acceleration pedal position $\phi_a$ during the transient operation 311 of the IC engine. For example, from points A to D in Fig. 4, 312 there are two tracks: A–B–D and A–C–D. Usually, when the 313 acceleration pedal changes, the throttle position follows, i.e., 314 the engine operating at track A–C–D. To make the IC engine 315 operate at the peak-efficiency curve L4, track A–B–D is better 316 to follow. However, a torque difference between the points 317 B and C occurs, and the electric motor is then employed to 318 generate a torque $T_{rm}$ to compensate it. In this paper, the motor 319 torque requirement $T_{rm}$ is determined through a torque split 320 factor $\lambda \in [-1, 1]$ as

$$T_{rm} = \lambda T_{dem}$$

(2)

where the value of $\lambda$ can be calculated by the following two 322 conditions: 1) braking and 2) driving.

**Braking:** During the braking, i.e., $T_{brk} > 0$, the split fac- 324 tor is expressed as follows:

$$\lambda = \begin{cases} 
0, & \text{ABS action} \\
1, & \text{Others.} 
\end{cases}$$

(3)

Based on (3), it can be known that the regeneration braking 326 will stop when the antilock brake system (ABS) is active, i.e., 327 wheels locking. For other conditions, e.g., $T_{brk} > T_{bmax}$, 328 the lacking braking torque is compensated by the hydraulic brake 329 system.

**Driving:** During the driving, $\lambda$ can also be computed 331 by the following two conditions: 1) recharging and 2) assist. 332 Recharging means that the SOC of the battery is lower than a 333 threshold and the battery needs to be recharging. Assist deno- 334 tes that the SOC is larger than the threshold and the battery can 335 output the electricity.

$$\lambda = \begin{cases} 
0, & \text{Kickoff for engine} \\
1, & \text{Kickoff for motor} \\
\frac{T_{dem} - T_{brk}}{T_{dem}}, & v_a < v_{th} \\
0, & v_a \geq v_{th} 
\end{cases}$$

(4)

where kickoff means that the switch at the end of acceleration 339 pedal is active.

In (4), the first item means that, when the kickoff switch is 341 active, the hybrid powertrain enters into the sport mode. The 342 second item means that, when the vehicle speed is low, the 343 hybrid powertrain enters into the pure-electric-driving mode. 344 The last item means that the powertrain enters into the hybrid- 345 driving mode.
Recharging: When the battery is recharging by using the electric motor, i.e., $T_{bat} > 0$, the split factor $\lambda$ is expressed as follows:

$$\lambda = \begin{cases} 0, & \text{Kickoff} \\ -\frac{T_{dem} - T_{pe}}{T_{dem} - T_{bat}}, & T_{dem} \geq T_{bat} + T_{pe} \\ -\frac{T_{bat} + T_{pe} - T_{dem}}{T_{bat}}, & T_{pe} < T_{dem} < T_{bat} + T_{pe} \\ -\frac{T_{bat} + T_{pe} - T_{dem}}{T_{bat}}, & T_{dem} \leq T_{pe}. \end{cases}$$ (5)

In (5), the first item denotes that the electric motor does not regenerate when the kickoff switch is active. The second, third, and fourth items are together to make the engine operate around the peak-efficiency curve. The last item denotes that, when $T_{dem} = 0$, the hybrid powertrain enters into the recharging mode. In fact, $\lambda$ has eight states of operation. The switched way of the torque split factor $\lambda$ among the eight states is described in Appendix A.

Then, the engine torque requirement $T_{re}$ is given by

$$T_{re} = \begin{cases} 0, & T_{brk} > 0 \\ 0, & T_{bat} = 0 \& v_a < v_{th} \\ T_{th}, & \text{Kickoff} \\ T_{pe}, & \text{Others}. \end{cases}$$ (6)

In (6), the first item describes that, during the brake, the IC engine stops. The second item describes that, when the vehicle speed is lower than the threshold $v_{th}$, the hybrid powertrain enters into the pure-electric-driving mode. The third item describes that, when the kickoff switch is active, the engine outputs the maximum power. The last item describes that the engine may operate around the peak-efficiency curve for other conditions.

Note that, in (6), it is shown that the target of the torque split is to make the engine operate around the peak-efficiency curve through the electric motor. The excess torque that is produced by the IC engine will be regenerated by the electric motor during the driving. The lacking torque between $T_{dem}$ and $T_{pe}$ is compensated by the electric motor.

The general torque demand is divided into the following two parts: 1) the engine torque requirement $T_{re}$ and 2) the motor torque requirement $T_{rm}$.

3) Torque Compensation: In fact, due to some nonlinear characteristics of engine dynamics, it is usually difficult for the engine torque demand to completely be realized by the MPC controller. Because the $q$-axis current of the motor is proportional to its output torque, we can suppose that the torque demand of the PMSM can completely be realized. Therefore, the torque error $\hat{T}_{err}$ between the engine torque demand $T_{de}$ and the actual torque $T_E$ can be compensated by the PMSM.

Here, a PD compensator is employed to compute the value of torque compensation $T_{cpt}$ as follows:

$$T_{cpt} = k_{pm} \hat{T}_{err} + k_{dm} \frac{d\hat{T}_{err}}{dt}.$$ (7)

In (7), the first formula utilizes the PD principle to compute the torque compensation $T_{cpt}$. In the second formula in (7), the estimated output torque of the IC engine $\hat{T}_E$ is calculated by a mean-value model, which is formulated in the next section.

Then, the motor torque request $T_{rm}$ is given as follows:

$$T_{rm} = T_{re} + T_{cpt}.$$ (8)

In (8), the lacking output torque that was derived from the nonlinearity of the IC engine dynamics is added into the torque request $T_{rm}$ of the electric motor.

4) Torque Limit: The output torque of the electric path, including the electric motor and battery, is limited by their own states, e.g., the SOC of the battery. Therefore, the torque request $T_{re}$ of the PMSM can be limited as follows:

$$T_{dM} = \begin{cases} \min\{T_{re}, T_{tmax}\}, & T_{re} \geq 0 \\ \max\{T_{re}, T_{tmin}\}, & T_{re} < 0. \end{cases}$$ (9)

In (9), the first formula shows a motor torque limit of the driving mode, and the second shows a motor torque limit of the braking mode.

Then, the engine torque demand $T_{de}$ can be revised as follows:

$$T_{de} = \begin{cases} T_{re} + T_{rm} - T_{dM}, & T_{brk} = 0 \\ T_{brk} > 0. \end{cases}$$ (10)

In (10), the first item denotes that the IC engine compensates for the lack of torque of electric path for the driving mode, i.e., $T_{brk} = 0$. However, the engine torque demand $T_{de}$ does not change for the braking mode, i.e., $T_{brk} > 0$.

Through the torque distributor, both the engine torque demand $T_{de}$ and the motor torque demand $T_{dm}$ are achieved and then transferred to the MPC controller of the engine and motor.

III. DESIGN OF CONTROLLER

As aforementioned, the torque demand control approach includes the following two MPC controllers: 1) the nonlinear MPC controller of the IC engine and 2) the linear MPC controller of the PMSM. A PI observer is also employed to estimate the torque load of the hybrid powertrain.

A. Nonlinear MPC Controller of the Engine

Due to the nonlinearity of engine dynamics, a nonlinear MPC controller will be designed by the mean-value model in this section.

1) Dynamic Model of the Engine: The mean-value model can be employed to compute the output torque of the IC engine by the intake manifold pressure $p_m$. The intake manifold dynamics, crankshaft dynamics, and combustion torque output $T_{ccht}$ are, respectively, given as follows [14]

$$\dot{\omega}_e = \frac{1}{\eta} (T_{ccht} - T_{los} - T_{le})$$

$$p_m = \frac{\eta_m Q_{th}R_{air}T_{air}u_{th}}{V_m}$$

$$T_{ccht} = \frac{\eta_m Q_{th}C_{tr}V_d}{4\pi R_{air}T_{air}} p_m.$$ (11)

Note that, in the aforementioned dynamic model of the 426 engine, the derivative of temperature is neglected, because the 427
intake manifold temperature is assumed constant. In general, the temperature derivative can be neglected, because it has only a minor effect on the manifold pressure dynamics.

2) State-Space Model: In this paper, it is supposed that the engine loss torque $T_{loss}$ within a small range of engine speed is similarly proportional to the engine speed $\omega$, based on a suitable factor $\eta_o$, i.e., $T_{loss} = \eta_o \omega$. As depicted in (11), we can choose the state variables as $x_1 = \omega$, $x_2 = p_m$, and the control input as $u = u_{th}$. Then, the engine model can be rewritten as the following state-space equation:

$$
\begin{align*}
\dot{x}_1 &= -a_1 x_1 + a_2 x_2 - d \\
\dot{x}_2 &= -a_3 x_1 x_2 + bu \\
y[k] &= cx_2[k]
\end{align*}
$$

whereas the parameters are given by $a_1 = \eta_e/I_d$, $a_2 = (\eta_e \eta_{cool} C_T V_d)/(4\pi I_d R_{air} T_{air})$, $a_3 = \eta_{cool} V_d/4\pi V_m$, $d = T_{Lc}/I_d$, and $b = (\eta_{cool} R_{air} T_{air})/V_m$.

3) Discrete-Time Model: Assume that the torque demand for the engine is delivered in discrete-time fashion with interval $T_s$, i.e., the desired reference signal for torque generation given at time $t = kT_s$ is $y[d][k+1]$, $y[a][k+2]$, ..., where $k$ denotes the sampling index with sampling period $T_s$. To design a desired control law in the discrete-time framework, we first make a one-step prediction of the system (13) through an additional one-step nonlinear MPC algorithm as depicted in (22) will be found. Substituting (14), (17) and (16) into (18), the cost function can be rewritten as follows:

$$
J(\Delta u[k]) = (r + w_1^2 h_1^2 + w_2^2 h_2^2) \Delta u^2[k] - 2c(w_1 \delta[k+1] + w_2 \delta[k+2]) \Delta u[k] + \dot{\delta}^2[k+1] + \dot{\delta}^2[k+2]
$$

where $\dot{\delta}[k+1] = y[d][k+1] - c(-a_2 x_1 x_2[k+1] + h_1 u[k-1])$

$$
\dot{\delta}[k+2] = y[d][k+2] - c(-a_2 x_1 x_2[k+2] + h_2 u[k-1])
$$

As denoted in (21), it is easy to see that the problem is to find the least value of the quadratic equation. Then, a straightforward calculation of the derivative to (21) yields the unique 480 optimal solution of cost function (18) as follows:

$$
\Delta u[k] = \frac{w_1 \delta x_1[k+1] + w_2 \delta x_2[k+2]}{r_1 + w_1^2 h_1^2 + w_2^2 h_2^2}
$$

Thus, based on the mean-value model, a torque-based two-step nonlinear MPC algorithm as depicted in (22) will be found. Based on the aforementioned deduction, it is shown that the 484 nonlinear MPC law has some characteristics as follows. First, as shown in (13), the mean-value model used is nonlinear. Second, as depicted in (22), the optimal solution of cost function is an 487 analytical expression, i.e., the control law is analytical. Finally, the optimal solution, as denoted in (22), is unique.

4) Nonlinear MPC Controller: Iteratively using this idea for the two-step-ahead prediction, under the condition that the control input keeps the value $u[k+1] = u[k]$, i.e., $\Delta u[k+1] = 0$, 486 yields

$$
\begin{align*}
\dot{x}_1[k+2] &= -a_3 a_2 x_1[k] x_2[k] + a_2^2 x_1[k] + p_1 x_2[k] - p_2 + a_3^2 x_2[k] \\
\dot{x}_2[k+2] &= -a_3 a_2 x_1[k] x_2[k] + p_1 x_2[k] + a_3^2 x_2[k] + a_3 a_2^2 x_1[k] x_2[k] + h_2 (u[k-1] + \Delta u[k])
\end{align*}
$$

where $p_1 = a_3 a_2 x_1[k] + a_2^2 x_1[k]$ and $h_2 = -a_3 a_2 x_1[k] x_2[k] - a_3 a_2^2 x_1[k] x_2[k] + a_3 a_2^2 x_1[k] + 2a_3 x_2[k]$. 486

The design problem of the control system considered can then be presented as follows. For given torque demand subsequence $y[d][k+1]$, $y[d][k+2]$, ..., find a feedback control law for each increment $\Delta u[k]$ of every predictive step. Solve the 486 following moving horizontal optimization problem [15]:

$$
\begin{align*}
\min_\Delta u[k] &\sum_{i=1}^{p} w_i (g[y[k+i|k] - y[d][k+i|k])^2 + \sum_{j=1}^{m} r_j \Delta u[k]^2 \\
\text{subject to} (14)-(17) &\text{and}
\end{align*}
$$

where $g[y[k+i|k] - y[d][k+i|k]$ are the predictions for the system output based 471 on the predictive states $x[k+i|k]$.

Now, we adopt a two-step prediction of system states and a one-step prediction of control input, i.e., $m = 1$, $p = 2$. 474 Substituting (14), (17) and (16) into (18), the cost function can be rewritten as follows:

$$
J(\Delta u[k]) = (r + w_1^2 h_1^2 + w_2^2 h_2^2) \Delta u^2[k] - 2c(w_1 \delta[k+1] + w_2 \delta[k+2]) \Delta u[k] + \dot{\delta}^2[k+1] + \dot{\delta}^2[k+2]
$$

As denoted in (21), it is easy to see that the problem is to find the least value of the quadratic equation. Then, a straightforward calculation of the derivative to (21) yields the unique 480 optimal solution of cost function (18) as follows:

$$
\Delta u[k] = \frac{w_1 \delta x_1[k+1] + w_2 \delta x_2[k+2]}{r_1 + w_1^2 h_1^2 + w_2^2 h_2^2}
$$

Thus, based on the mean-value model, a torque-based two-step nonlinear MPC algorithm as depicted in (22) will be found. Based on the aforementioned deduction, it is shown that the 484 nonlinear MPC law has some characteristics as follows. First, as shown in (13), the mean-value model used is nonlinear. Second, as depicted in (22), the optimal solution of cost function is an 487 analytical expression, i.e., the control law is analytical. Finally, the optimal solution, as denoted in (22), is unique.

B. Linear MPC Controller of the PMSM

A linear MPC controller is employed to predict the current and torque behavior through a mathematical model of the PMSM. The electrical dynamics of PMSM is given as follows: 492

$$
\begin{align*}
\frac{d}{dt} I_d &= \frac{1}{\tau_m} (u_d - R_m I_d + \omega_m L_m i_q) \\
\frac{d}{dt} I_q &= \frac{1}{\tau_m} (u_q - R_m I_q - \omega_m L_m i_d - \omega_m \Psi_m)
\end{align*}
$$
Assume that the rotor speed is constant in the predictive 501 period, which indicates that \( \omega_{me} \) is a constant. Then, the voltage vector is selected as the control input \( U_u = [i_d, i_q]^T \), and the current vector is selected as state variables \( X_i = [i_d, i_q]^T \) on the \( d \) - and \( q \)-axes. With the aforementioned choice, the discrete-time model with sampling time \( T_s \) at the time \( k \) is given as follows:

\[
X_i[k+1|k] = AX_i[k|k] + BU_u[k|k] + E
\]  

(24)

where

\[ A = T_s \begin{bmatrix} -\frac{R}{L} + 1 & \omega_{me} \\ \omega_{me} & -\frac{R}{L} + 1 \end{bmatrix}, \quad B = T_s/L \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad E = \begin{bmatrix} 0 \\ \omega_{Lm}\Psi_m \end{bmatrix}, \]

and \( U_u[k|k] \) is used as the control input at time

\[ U_u[k|k] = U_u[k-1] + \Delta U_u[k|k] \]  

(25)

where \( U_u[k-1] \), the control input of the previous step, is known at time \( k \).

Based on the predictive current of the \( d \)- and \( q \)-axes in (24), the torque behavior of the PMSM in the future is predicted as follows:

\[ T_m[k|k] = C_m i_q[k|k]. \]  

(26)

Furthermore, let the torque demand of the electric motor be \( T_{dM} \). Because the number of predictive steps is equivalent to \( n \) for both the control input \( U_u \) and the system output \( \hat{T}_m \), for the predictive subsequence of the PMSM output torque \( \hat{T}_m[k+1|k] \), the cost function of the electric motor is given as follows:

\[
\Delta U_u[k|k] \rightarrow \min \left\{ \sum_{j=1}^{n} \lambda_{mj} \left( \hat{T}_m[k+j|k] - T_{dM}[k+j|k] \right)^2 \right\} + \sum_{j=1}^{n} \rho_{mj} \Delta U_u[k|k] \]  

(27)

subject to (24)–(26), and

\[
\begin{align*}
|\Delta U_u[k|k]| & \leq U_{umax} \\
|U_u[k]| & \leq U_{umax} \\
i_{dmin} & \leq X_1[k|k] \leq i_{dmax} \\
i_{qmin} & \leq X_2[k|k] \leq i_{qmax}.
\end{align*}
\]  

(28)

At time \( k \), substituting (24) and (26) into the cost function (27), it is straightforward to achieve the optimal control voltage increment \( \Delta U_u[k|k] \) of the PMSM \( d \)- and \( q \)-axes for the optimal output to inverter as in (30), shown at the bottom of the page, where \( z = 1 \ldots n \).

As depicted in (30), the solution is a subsequence. However, the first term \( \Delta U_u[k|k] \) is used as the control input increment, and then, achieving the optimal control input, \( U_u[k|k] \) is applied to control the PMSM at time \( k \). This process is repeated every sampling time.

IV. DESIGN OF THE TORQUE LOAD OBSERVER

In practice, the torque load of the hybrid powertrain is an immeasurable variable in engineering. In the following context, a PI observer is designed to estimate the torque load of the IC engine.

Let the engine output torque \( T_e = T_{cht} - T_{los} \). Then, the crankshaft dynamics of (11) can be rewritten as follows:

\[
I_e \ddot{\omega}_e = T_e - T_{Le}. \]

(31)

Assumption 1: At time \( k+1 \), the load torque is \( T_{L0} \). Then, at time \( k \), the actual load \( T_{Le} \) can be described as follows:

\[
T_{Le} = T_{L0} + \Delta(\dot{\omega}_e) \]  

(32)

where \( \Delta(\dot{\omega}_e) \) is an unknown function that is bounded by a positive constant \( \zeta(\dot{\omega}_e) \), i.e., \( |\Delta(\dot{\omega}_e)| \leq \zeta(\dot{\omega}_e) \). Based on the principle of the PI observer in Fig. 5, the load observer can be formulated as follows:

\[
\begin{align*}
\hat{T}_{Le} & = k_p \dot{\omega}_e + k_i \int \dot{\omega}_e \, dt + \zeta(\dot{\omega}_e) \text{sgn}(\dot{\omega}_e) \\
\dot{\omega}_e & = \frac{1}{I_a} (T_e - \hat{T}_{Le})
\end{align*}
\]  

(33)

where \( \text{sgn} \) is a signum function.

Note that, when the observer is applied to engineering practice by digital implementation, the estimated error may not tend to zero due to the limited sampling rate and quantization.

Let \( \sigma = k_i \int \dot{\omega}_e \, dt \). Combining (31)–(33), the error equation can be given as follows:

\[
\begin{align*}
I_a \dot{\omega}_e & = \Delta(\dot{\omega}_e) + T_{L0} - k_p \dot{\omega}_e - \sigma - \zeta(\dot{\omega}_e) \text{sgn}(\dot{\omega}_e) \\
\dot{\sigma} & = k_i \dot{\omega}_e.
\end{align*}
\]  

(34)

Theorem 1: Suppose that Assumption 1 holds. Then, for any positive constant \( k_p \) and \( K_i \), the error system (34) is Lyapunov stable.

Proof: Let \( \dot{\sigma} = T_{L0} - \sigma \). The error dynamics in (34) are rewritten as

\[
\begin{align*}
I_a \dot{\omega}_e & = \Delta(\dot{\omega}_e) - k_p \dot{\omega}_e + \sigma - \zeta(\dot{\omega}_e) \text{sgn}(\dot{\omega}_e) \\
\dot{\sigma} & = k_i \dot{\omega}_e.
\end{align*}
\]  

(35)
Consider the candidate of Lyapunov function as

\[ V = \frac{1}{2} \left( I_d \dot{\omega}_e^2 + \frac{1}{k_i} \sigma^2 \right). \]  

Then

\[ \dot{V} = (\Delta(\dot{\omega}_e) - \xi(\dot{\omega}_e)\text{sgn}(\dot{\omega}_e)) \dot{\omega}_e - k_p \dot{\omega}_e^2. \]  

Note that \( |\Delta(\dot{\omega}_e)| \leq \xi(\dot{\omega}_e) \); therefore

\[ \dot{V} \leq 0. \]  

Equation (38) denotes that the error system is of Lyapunov stability. Furthermore, combining (35)–(38), it can be concluded that

\[ \dot{\omega}_e = 0 \iff \dot{\omega}_e = 0, \quad \sigma = 0, \quad T_{L0} + \Delta = \sigma. \]  

Based on the principle of LaSalle’s invariant set, \( \omega_e \) and \( \sigma \) converge to the set where \( \dot{\omega}_e(\sigma) = (0, -\Delta) \) as \( t \to \infty \). Hence, combining (32) and (33) in the set \( \omega_e = 0 \) and \( T_{L0} + \Delta = \sigma \), the estimated load torque is given as follows:

\[ \dot{T}_{Le} = \sigma = T_{L0} + \Delta. \]  

Equation (40) implies that, in the invariant set, the estimated load torque \( \dot{T}_{Le} \to T_{Le} \) as \( t \to \infty \).

Remark 1: The aforementioned analysis does not consider the chattering from the switching of the signum function. A modified signum function \( \text{sgn}_m(\dot{\omega}_e) \) can attenuate the chattering in the practical application, which is given as follows:

\[ \text{sgn}_m(\dot{\omega}_e) = \begin{cases} \dot{\omega}_e \leq B_{th} & \omega_e \neq 0 \\ \frac{\dot{\omega}_e}{B_{th}} & |\dot{\omega}_e| > B_{th}. \end{cases} \]  

With regard to the proportional and integral gains \( k_p, K_i \) of the torque load observer, the different powertrain needs different values. Li and his cooperators describe the tuning rules of PI gains in detail in [16].

V. SIMULATION VALIDATION

To verify the MPC approach of the parallel hybrid powertrain, some simulation scenarios are carried out in MATLAB/Simulink.

A. Description of the Simulation

The simulation platform of the parallel hybrid powertrain is constructed through MATLAB/Simulink. The parameters of the simulation model, including the engine, PMSM, and vehicle, are shown in the Appendix B.

In the following cases of simulation, the MPC algorithm of engine uses a two-step prediction of state variables, a one-step prediction of control input, and a one-step desired torque. The weights of the cost function for the PMSM are \( \rho_m = 0.72 \), \( \lambda_m = 1 \).

B. Simulation Results

Two operating scenarios that consist of the step throttle and step load are tested on the simulation platform that was developed 591 for the parallel hybrid powertrain. The scenario of the step throttle simulates a driver giving a step torque demand during 593 the normal operation to accelerate the vehicle. The scenario of the step throttle simulates a driver abruptly turning on a power 595 accessory, e.g., air conditioner.

1) Case 1—Step Throttle With Hybrid and Conventional Powertrains: The simulation scenario of the step throttle is employed to simulate the step increment of the acceleration 599 pedal from the driver. The aim of this scenario is to validate the torque demand controller with the PD compensation and 601 PI observer. In the following context, two compared simulation 602 results are illustrated and analyzed. Both results employ the 603 torque demand controller, but one result aims at the hybrid 604 powertrain, whereas the other result aims at the conventional 605 powertrain.

Fig. 6 shows some torque signals of the step throttle simulation. Fig. 6(a) shows some simulation results of the engine torque for the hybrid powertrain. In this figure, the pre-dictive torque from the MPC controller can follow the engine 610 torque demand \( T_{Le} \). However, it is obvious that there is an error 611 between the engine torque demand and the actual engine torque 612 due to the error of the mean-value model of the IC engine.

After 19 s, as shown in Fig. 6, a step increment of the torque 614 demand of about 50 Nm is applied to the powertrain. It is shown 615 that there is a response time delay between the predictive engine 616 torque and the actual torque. For the time delay, the motor- 617 assisted hybrid powertrain can provide an equivalent torque 618 compensation, as illustrated in Fig. 6(c), by using the electric 619...
Fig. 7. General torque of the step throttle scenario. HD means the general torque demand of the hybrid powertrain. HA means the actual general torque of the hybrid powertrain. CD means the general torque demand of the conventional powertrain. CA means the actual general torque of the conventional powertrain.

From 19.1 s to 19.4 s in Fig. 6(c), it is shown that the PMSM can output a torque to compensate for the hybrid powertrain at the initial phase of the step throttle. Compared with Fig. 6(a) and (c), it is shown that, after finishing the shift from gear 2 to gear 3, the MPC controller with the PD compensator can provide a torque compensation to make up for the time delay of the torque response of the IC engine.

Fig. 7 shows some simulation results of the general torque. For the hybrid powertrain, the general torque is the sum of the engine and motor torque, i.e., \( T_E + T_M \). For the conventional powertrain, the general torque is only the engine torque \( T_E \). In this figure, it is shown that the electric motor can regenerate the redundant torque of the IC engine between the torque demand and the actual torque from 19 s to 21 s. However, it is impossible for the conventional powertrain to do it. Hence, the IC engine can operate around the peak-efficiency curve by using the electric motor.

Fig. 8 shows some simulation results of the torque engine throttle, speed, and \( q \)-axis current of the electric motor. In Fig. 8(a), it is shown that the shifting of the hybrid powertrain lags behind that of the conventional because the MPC controller with the PD compensator regenerates the excess torque to the engine torque demand \( T_{dE} \). From 19.5 s to 21.2 s in Fig. 8, due to the PD compensation, the PMSM absorbs the excess torque of the powertrain, which results in the lag of shifting for the hybrid powertrain.

After engagement of the synchronizer from 22 s to 22.5 s, the engine speed of the hybrid powertrain is larger than the conventional due to the behavior of the compensated torque. Comparing Figs. 6(c) and 8(c), it is shown that the motor torque is proportional to the motor current at the \( q \)-axis. In Fig. 8(a), it is easy to figure out that the consumption of fuel is almost the same between the hybrid powertrain and the conventional.

Fig. 9 shows some simulation results of the clutch, gear position, and vehicle speed. In the simulation, the vehicle speed is calculated by the longitudinal-dynamics formula, in which the driving resistances include the braking force, air resistance, slope resistance, and roll resistance. In Fig. 9(a) and (b), a shift from gear 2 to gear 3 occurs from 21 s to 22 s for the hybrid powertrain and the conventional, respectively. In Fig. 9(c), the resulting vehicle speed still exceeds that of the conventional powertrain due to the torque compensation after the synchronization.

Based on Fig. 9(c), at 19.3 and 22.3 s, the torque compensation from the electric motor is beneficial to the acceleration of the vehicle, which makes the vehicle speed of the hybrid powertrain exceed that of the conventional. Due to the energy recovery from 19.5 s to 21.2 s, the PMSM may not cost additional electric energy.
In summary, based on the aforementioned analysis, it can be concluded that the torque demand controller with the PD compensator fits the hybrid powertrain.

2) Case 2—Step Load With Hybrid and Conventional Powertrains: The simulation scenario of the step load is employed to simulate a driver suddenly turning on a vehicle accessory, e.g., air conditioner, during normal driving. The aim of this scenario is to validate that the torque demand control strategy may diminish the effect of the step torque load. In the following context, two compared simulation results from both the hybrid powertrain and the conventional are illustrated and analyzed.

Fig. 10 shows some simulation results of the demand, actual, and predictive engine torque and the additional torque load of the powertrain. The predictive engine torque is from the MPC controller. Likewise, the predictive engine torque corresponds with the engine torque demand. Due to the error of the mean-value model, there is a difference between the predictive engine torque and the actual.

For the step load scenario, the engine torque demand is 150 Nm during the whole operation. At 10.7 s, a step torque of 50 Nm loaded on the powertrain. Compared with Fig. 10(a) and (b), it is shown that the shifting time of the hybrid powertrain advanced that of the conventional by about 1 s due to the torque compensation from the motor, as depicted in Fig. 10(c), from 10.7 s to 12.5 s. After synchronization of clutch, the PMSM outputs a torque to compensate for the slower torque response of the IC engine. When the torque demand controller has made sure that the negative effect from step torque load was diminished, the torque compensation from the PMSM is released.

Fig. 11 shows some simulation results of the general torque. Likewise, the general torque is the sum of the engine and motor torque $T_E + T_M$ for the hybrid powertrain. For the conventional powertrain, the general torque is only the engine torque $T_E$. From 10.7 s to 12.5 s, as shown in Fig. 11, the general torque demands significantly differ, irrespective of the same throttle commands and vehicle speed traces because the hybrid powertrain can utilize the electric motor to compensate for a torque of the step load, i.e., the step load from vehicle accessories can rapidly be compensated. However, the conventional powertrain cannot compensate for it, except for drivers changing the acceleration pedal position $\phi_a$.

Fig. 12 shows some simulation results of the engine throttle, speed, and motor current at the $q$-axis. Fig. 12(a) shows that the consumption of fuel is also almost the same between the hybrid and the conventional powertrain. In Fig. 12(b), it is shown that the engine speed of the hybrid powertrain is larger than the conventional. Comparing Figs. 12(c) and 10(c), likewise, it is shown that the motor torque is proportional to the motor current at the $q$-axis.

Fig. 13 shows some simulation results of the clutch, gear position, and vehicle speed. In Fig. 13(a), the shifting time of the conventional powertrain lags behind that of the hybrid for about 1 s. In Fig. 13(a), the shift is from gear 2 to gear 3.
Fig. 13. Vehicle states of the step load scenario.

An MPC-based torque demand control approach has been developed to implement the torque-based control of a parallel hybrid powertrain that consists of a torque distributor, a nonlinear MPC controller, a linear MPC controller of the PMSM, and a torque load observer. A torque distributor with the PD compensator is designed to make the IC engine operate around the peak-efficiency curve. The PD compensator can compensate for the torque error between the engine torque demand and the actual engine torque during normal operation.

A PI observer is designed for the torque demand controller to estimate the torque load of the hybrid powertrain, which is Lyapunov stable. Compared with the conventional method, calculating the driving resistance by vehicle speed, the PI observer has a better dynamic performance.

### Appendix A

#### Torque Split Factor

<table>
<thead>
<tr>
<th>Value of $\lambda$</th>
<th>$T_{dem}$</th>
<th>Type of operation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda = -1$</td>
<td>Positive</td>
<td>Maximum recharging using engine</td>
</tr>
<tr>
<td>$\lambda \in (-1, 0)$</td>
<td>Positive</td>
<td>Partial recharging using engine</td>
</tr>
<tr>
<td>$\lambda = 0$</td>
<td>Positive</td>
<td>Provide all torque using engine</td>
</tr>
<tr>
<td>$\lambda \in (0, 1)$</td>
<td>Negative</td>
<td>Purely conventional braking</td>
</tr>
<tr>
<td>$\lambda = 1$</td>
<td>Positive</td>
<td>Provide all torque using motor</td>
</tr>
<tr>
<td>$\lambda = -1$</td>
<td>Negative</td>
<td>Maximum regenerative braking</td>
</tr>
</tbody>
</table>

In this paper, it has been assumed that the torque generation of the PMSM is proportional to the current of the $q$-axis. Therefore, the PD compensator of the torque demand controller should be employed to compensate for the nonlinear torque error of the general torque demand instead of the IC engine torque error. In addition, the torque demand control approach will be validated in the test bench of a parallel hybrid powertrain and even on a vehicle in the future.

### Appendix B

#### Parameters of the Simulation Platform for the Parallel Hybrid Powertrain

The parameters used for the simulation platform for the parallel hybrid powertrain are listed as follows.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type</td>
<td>V6</td>
</tr>
<tr>
<td>Injection</td>
<td>Port injection.</td>
</tr>
<tr>
<td>Displacement</td>
<td>2.9 L.</td>
</tr>
<tr>
<td>Maximum Power</td>
<td>189 kW/(6400 r/min).</td>
</tr>
<tr>
<td>Maximum Torque</td>
<td>300 Nm/(4800 r/min).</td>
</tr>
<tr>
<td>Stator inductance</td>
<td>0.00015 H.</td>
</tr>
<tr>
<td>Stator resistance</td>
<td>0.01 $\Omega$.</td>
</tr>
<tr>
<td>Flux induced by the magnet</td>
<td>0.12 Wb.</td>
</tr>
<tr>
<td>Pole pairs</td>
<td>4.</td>
</tr>
<tr>
<td>Motor inertia</td>
<td>0.4 kg/m$^2$.</td>
</tr>
<tr>
<td>Vehicle cross section</td>
<td>2 m$^2$.</td>
</tr>
<tr>
<td>$C_w$-value of the vehicle</td>
<td>0.3.</td>
</tr>
<tr>
<td>Vehicle mass</td>
<td>1250 kg.</td>
</tr>
<tr>
<td>Tire-road friction coefficient</td>
<td>0.01.</td>
</tr>
<tr>
<td>Dynamic tire radius</td>
<td>0.35 m.</td>
</tr>
<tr>
<td>Differential transmission ratio</td>
<td>3.7.</td>
</tr>
<tr>
<td>Transmission</td>
<td>6 Gear.</td>
</tr>
<tr>
<td>Reverse-gear ratio</td>
<td>3.5.</td>
</tr>
</tbody>
</table>

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REFERENCES


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