

1. TAYLOR 展開の例

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \dots \quad (|x| < 1)$$

$$\frac{1}{(1-x)^2} = \sum_{n=0}^{\infty} (n+1)x^n = 1 + 2x + 3x^2 + 4x^3 + \dots \quad (|x| < 1)$$

$$(1+x)^\alpha = \sum_{n=0}^{\infty} \binom{\alpha}{n} x^n = 1 + \alpha x + \frac{\alpha(\alpha-1)}{2} x^2 + \frac{\alpha(\alpha-1)(\alpha-2)}{3!} x^3 + \dots \quad (|x| < 1)$$

$$\sqrt{1+x} = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 - \frac{5}{128}x^4 + \dots \quad (|x| < 1)$$

$$\frac{1}{\sqrt{1-x}} = 1 + \frac{1}{2}x + \frac{3}{8}x^2 + \frac{5}{16}x^3 + \frac{35}{128}x^4 + \dots \quad (|x| < 1)$$

$$\log(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} x^n = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \dots \quad (|x| < 1)$$

$$\log \frac{1}{1-x} = \sum_{n=1}^{\infty} \frac{1}{n} x^n = x + \frac{1}{2}x^2 + \frac{1}{3}x^3 + \frac{1}{4}x^4 + \dots \quad (|x| < 1)$$

$$e^x = \exp x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \dots$$

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1} = x - \frac{1}{6}x^3 + \frac{1}{120}x^5 - \frac{1}{5040}x^7 + \dots$$

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n} = 1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 - \frac{1}{720}x^6 + \dots$$

$$\sinh x = \sum_{n=0}^{\infty} \frac{1}{(2n+1)!} x^{2n+1} = x + \frac{1}{6}x^3 + \frac{1}{120}x^5 + \frac{1}{5040}x^7 + \dots$$

$$\cosh x = \sum_{n=0}^{\infty} \frac{1}{(2n)!} x^{2n} = 1 + \frac{1}{2}x^2 + \frac{1}{24}x^4 + \frac{1}{720}x^6 + \dots$$

$$\arcsin x = \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} \binom{-\frac{1}{2}}{n} x^{2n+1} = x + \frac{1}{6}x^3 + \frac{3}{40}x^5 + \frac{5}{112}x^7 + \dots \quad (|x| < 1)$$

$$\arctan x = \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} x^{2n+1} = x - \frac{1}{3}x^3 + \frac{1}{5}x^5 - \frac{1}{7}x^7 + \dots \quad (|x| < 1)$$

1-1. 演習問題. 次の関数の Taylor 展開を、例えば x^4 の項まで (出来ればもっと) 求めよ。

(1) $e^{-x^2} = \exp(-x^2)$ (e^x の Taylor 展開に $-x^2$ を代入せよ)

(2) $e^x \cos x$ (e^x の展開と $\cos x$ の展開とを掛け算せよ)

(3) $\frac{1}{1-x-x^2}$ (筆算の割り算の要領で計算せよ。係数の列に見覚えは?)

(4) $\frac{1}{\cos x}$ (分母が冪級数でも同様に割り算の計算ができる)

(5) $-\log(1-x)$ ($\frac{1}{1-x}$ の展開の両辺を積分せよ)

以下はノーヒント。いろいろ試みよ。

(6) $e^{x+x^2} = \exp(x+x^2)$

(7) $\frac{1}{(1-x)(1-2x)}$