1. 不等式・ε-δ 論法

1–1. **不等式の基本性質.**

- $x \leq y, y \leq z \Longrightarrow x \leq z$: **推移律 (transitive law)**
- $x \leq y, y \leq x \Longrightarrow x = y$: **反対称律 (anti-symmetric law)**
- 演算との関係 :
  - $\star$ $x \leq y \Longrightarrow x + a \leq y + a$
  - $\star$ $a > 0, x \leq y \Longrightarrow ax \leq ay$
- $|x + y| \leq |x| + |y|$: **三角不等式 (triangle inequality)**

1–2. ε-δ **式の極限の定式化.**

- 関数 f に対し、$x \longrightarrow a$ のとき $f(x) \longrightarrow b$
  ($f(x)$ が b に**収束 (converge)** する, $\lim\limits_{x \to a} f(x) = b$)
  $\overset{\leftarrow}{\Longleftrightarrow} \forall \varepsilon > 0 : \exists \delta > 0 : 0 < |x - a| < \delta \Longrightarrow |f(x) - b| < \varepsilon$
- 関数 f が $x = a$ で**連続 (continuous)** $\overset{\leftarrow}{\Longleftrightarrow} \lim\limits_{x \to a} f(x) = f(a)$
  $\Longleftrightarrow \forall \varepsilon > 0 : \exists \delta > 0 : |x - a| < \delta \Longrightarrow |f(x) - f(a)| < \varepsilon$

2. Taylor 展開の例

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \cdots \quad (|x| < 1)$$

$$\frac{1}{(1-x)^2} = \sum_{n=0}^{\infty} (n+1) x^n = 1 + 2x + 3x^2 + 4x^3 + \cdots \quad (|x| < 1)$$

$$(1+x)^{\alpha} = \sum_{n=0}^{\infty} \binom{\alpha}{n} x^n = 1 + \alpha x + \frac{\alpha(\alpha-1)}{2} x^2 + \frac{\alpha(\alpha-1)(\alpha-2)}{3!} x^3 + \cdots \quad (|x| < 1)$$

$$\sqrt{1+x} = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 - \frac{5}{128}x^4 + \cdots \quad (|x| < 1)$$

$$\frac{1}{\sqrt{1-x}} = 1 + \frac{1}{2}x + \frac{3}{8}x^2 + \frac{5}{16}x^3 + \frac{35}{128}x^4 + \cdots \quad (|x| < 1)$$

$$\log(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} x^n = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \cdots \quad (|x| < 1)$$

$$\log \frac{1}{1-x} = \sum_{n=1}^{\infty} \frac{1}{n} x^n = x + \frac{1}{2}x^2 + \frac{1}{3}x^3 + \frac{1}{4}x^4 + \cdots \quad (|x| < 1)$$

$$e^x = \exp x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \cdots$$

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1} = x - \frac{1}{6}x^3 + \frac{1}{120}x^5 - \frac{1}{5040}x^7 + \cdots$$

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n} = 1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 - \frac{1}{720}x^6 + \cdots$$