

9. 積分の公式

$$\begin{array}{ll}
 \int x^\alpha dx = \frac{1}{\alpha+1} x^{\alpha+1} \quad (\alpha \neq -1) & \int \sec^2 x dx = \tan x \\
 \int \frac{dx}{x} = \log |x| & \int \operatorname{cosec}^2 x dx = -\cot x \\
 \int \frac{f'(x)}{f(x)} dx = \log |f(x)| & \int \frac{dx}{\sqrt{1-x^2}} = \begin{cases} \arcsin x \\ -\arccos x \end{cases} \quad (\text{注}) \\
 \int e^x dx = e^x & \int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin \frac{x}{|a|} \\
 \int a^x dx = \frac{a^x}{\log a} \quad (a > 0, a \neq 1) & \int \frac{dx}{1+x^2} = \arctan x \\
 \int f'(x)e^{f(x)} dx = e^{f(x)} & \int \frac{dx}{a^2+x^2} = \frac{1}{a} \arctan \frac{x}{a} \\
 \int \log x dx = x \log x - x & \int \frac{dx}{\sqrt{1+x^2}} = \operatorname{arcsinh} x = \log(x + \sqrt{1+x^2}) \\
 \int \sin x dx = -\cos x & \int \frac{dx}{\sqrt{a+x^2}} = \log |x + \sqrt{a+x^2}| \\
 \int \cos x dx = \sin x & \int \frac{dx}{1-x^2} = \operatorname{arctanh} x = \frac{1}{2} \log \left| \frac{1+x}{1-x} \right| \\
 \int \tan x dx = -\log |\cos x| & \int \sqrt{1+x^2} dx = \frac{1}{2} (x\sqrt{1+x^2} + \log(x + \sqrt{1+x^2})) \\
 \int \cot x dx = \log |\sin x| & \int \sqrt{x^2-1} dx = \frac{1}{2} (x\sqrt{x^2-1} - \log |x + \sqrt{x^2-1}|) \\
 \int \sinh x dx = \cosh x & \int \arcsin x dx = x \arcsin x + \sqrt{1-x^2} \\
 \int \cosh x dx = \sinh x & \int \arctan x dx = x \arctan x - \frac{1}{2} \log(1+x^2) \\
 \int \tanh x dx = \log \cosh x & \int \operatorname{arcsinh} x dx = x \operatorname{arcsinh} x - \sqrt{1+x^2} \\
 \int \operatorname{coth} x dx = \log |\sinh x| & \int \operatorname{arctanh} x dx = x \operatorname{arctanh} x + \frac{1}{2} \log(1-x^2)
 \end{array}$$

(注)

- $\arcsin x = \frac{\pi}{2} - \arccos x$ ではあるが、不定積分では定数の差は気にしないので、いづれでもよい。
- $\arcsin x, \arctan x$ 等は所謂主値を取る： $-\frac{\pi}{2} \leq \arcsin x \leq \frac{\pi}{2}, -\frac{\pi}{2} < \arctan x < \frac{\pi}{2}$

— 例示は理解の試金石
結城 浩「数学ガール」(<http://www.hyuki.com/girl/>) より