9. 積分の公式

$$\int x^\alpha dx = \frac{1}{\alpha+1}x^{\alpha+1} \quad (\alpha \neq 1)$$
$$\int \frac{dx}{x} = \log|x|$$
$$\int \frac{f'(x)}{f(x)}dx = \log|f(x)|$$
$$\int e^x dx = e^x$$
$$\int a^x dx = \frac{a^x}{\log a} \quad (a>0, a\neq 1)$$
$$\int f'(x)e^{f(x)}dx = e^{f(x)}$$
$$\int \log x dx = x\log x - x$$
$$\int \sin x dx = -\cos x$$
$$\int \cos x dx = \sin x$$
$$\int \tan x dx = -\log|\cos x|$$
$$\int \cot x dx = \log|\sin x|$$
$$\int \sinh x dx = \cosh x$$
$$\int \cosh x dx = \sinh x$$
$$\int \tanh x dx = \log\cosh x$$
$$\int \coth x dx = \log|\sinh x|$$

$$\int \sec^2 x dx = \tan x$$
$$\int \mathrm{cosec}^2 x dx = -\cot x$$
$$\int \frac{dx}{\sqrt{1-x^2}} = \begin{cases} \arcsin x \\ -\arccos x \end{cases} \quad (\text{注})$$
$$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{|a|}$$
$$\int \frac{dx}{1+x^2} = \arctan x$$
$$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a}$$
$$\int \frac{dx}{\sqrt{1+x^2}} = \mathrm{arcsinh}\, x = \log(x+\sqrt{1+x^2})$$
$$\int \frac{dx}{\sqrt{a+x^2}} = \log|x+\sqrt{a+x^2}|$$
$$\int \frac{dx}{1-x^2} = \mathrm{arctanh}\, x = \frac{1}{2}\log\left|\frac{1+x}{1-x}\right|$$
$$\int \sqrt{1+x^2}dx = \frac{1}{2}\left(x\sqrt{1+x^2}+\log(x+\sqrt{1+x^2})\right)$$
$$\int \sqrt{x^2-1}dx = \frac{1}{2}\left(x\sqrt{x^2-1}-\log|x+\sqrt{x^2-1}|\right)$$
$$\int \arcsin x dx = x\arcsin x + \sqrt{1-x^2}$$
$$\int \arctan x dx = x\arctan x - \frac{1}{2}\log(1+x^2)$$
$$\int \mathrm{arcsinh}\, x dx = x\,\mathrm{arcsinh}\, x - \sqrt{1+x^2}$$
$$\int \mathrm{arctanh}\, x dx = x\,\mathrm{arctanh}\, x + \frac{1}{2}\log(1-x^2)$$

（注）

- $\arcsin x = \frac{\pi}{2} - \arccos x$ ではあるが、不定積分では定数の差は気にしないので、いづれでもよい。
- $\arcsin x, \arctan x$ 等は所謂主値を取る：$-\frac{\pi}{2} \leq \arcsin x \leq \frac{\pi}{2}, -\frac{\pi}{2} < \arctan x < \frac{\pi}{2}$

— 例示は理解の試金石
結城 浩「数学ガール」(http://www.hyuki.com/girl/) より