

1 Water-Quality Trading:
2 Can We Get the Prices of Pollution Right?

3 Yoshifumi Konishi

4 Faculty of Liberal Arts, Sophia University,
5 7-1 Kioi-cho, Chiyoda-ku Tokyo 102-8554 Japan

6 Jay S. Coggins

7 Department of Applied Economics, University of Minnesota
8 1994 Buford Avenue, St. Paul MN 55108

9 Bin Wang

10 Department of Pediatrics, University of Chicago
11 900 East 57th Street, KCBD 5121, Chicago, IL 60637

12 Draft: March 7, 2014

13 Revised: March 6, 2015

14 Forthcoming, *Water Resources Research*

15 Konishi: Associate Professor, Sophia University. Coggins: Associate Professor, University of
16 Minnesota. Wang, Department of Pediatrics, University of Chicago. We gratefully acknowledge
17 financial support from a U.S. EPA 2005 Targeted Watershed Grant and from the Japan Society
18 for the Promotion of Science Grants-in-Aid for Scientific Research (Grant Numbers: 23730250 and
19 25630208). We also thank Werner Antweiler, Scott Farrow, Qiuqiong Huang, Frances Homans,
20 Jacob LaRiviere, and seminar participants at Kyoto University, the University of Minnesota, the
21 2nd Congress of the East Asian Association of Environmental and Resource Economics, the 2012
22 CREE meetings, and the 2013 AERE meetings for their helpful comments.

Abstract

24 Water-quality trading requires inducing permit prices that account properly for spatially explicit
25 damage relationships. We compare recent work by Hung and Shaw (2005) and Farrow *et al.* (2005)
26 for river systems exhibiting branching and nonlinear damages. The Hung-Shaw scheme is robust to
27 nonlinear damages, but not to hot spots occurring at the confluence of two branches. The Farrow *et*
28 *al.* scheme is robust to branching, but not to nonlinear damages. We also compare the two schemes
29 to each other. Neither dominates from a welfare perspective, but the comparison appears to tilt in
30 favor of the Farrow *et al.* scheme.

31 KEYWORDS: Water-quality trading, trading ratios, branching, nonlinear damages, spatially explicit
32 prices

1 Introduction

34 Enthusiasm for water-quality trading (WQT) is high in the U.S., at least in certain quarters (U.S.
35 EPA, 2003, 2004). The experience with sulfur dioxide allowances proved that markets can work for
36 air pollution. Should they not work for water pollution too? So far, where trade in water quality
37 has been attempted the results have not been encouraging, at least according to one important
38 metric: the number of trades has typically been lower than hoped (Morgan and Wolverton 2005,
39 King and Kuch 2003, Fisher-Vanden and Olmstead 2013). Is this because trading is simply not a
40 viable policy approach for water, or because the existing schemes are not well designed?

41 Hoping that the problem is with design, not with concept, economists continue to seek new
42 and improved ways of building WQT systems. The aim is to improve their performance and so,
43 perhaps, to achieve the kind of cost savings that accompanied trading for SO₂ under Title IV of
44 the 1990 Clean Air Act Amendments (Carlson *et al.* 2000). Two recent contributions, by Farrow *et*
45 *al.* (2005) and by Hung and Shaw (2005), follow this strategy. Though quite different in specifics,
46 both offer innovative schemes for trading between point sources on a river system.

47 The present paper grew out of our efforts to employ these two approaches in a study of trading
48 for temperature in the Vermillion River, a popular trout stream in suburban Minnesota (Vermillion
49 River Watershed 2008). In each case we encountered difficulties that could be traced to particular
50 features of the respective schemes. The Vermillion trout fishery extends into tributaries, and so the
51 trading system we sought would need to accommodate branching. The biology of trout indicates
52 the existence of threshold temperatures above which the fish are dramatically more susceptible to
53 mortality (Elliott 2000). The trading system we sought would need to accommodate nonlinear
54 damages.

55 Our attempt to apply Hung and Shaw revealed a problem in their model that arises when a “hot
56 spot” is located at a confluence of streams. There, the Hung-Shaw method of allocating permits is
57 ill defined. Our attempt to apply Farrow *et al.* revealed a problem in their model that arises when
58 the damages associated with emissions are nonlinear. There, the Farrow *et al.* damage coefficients
59 cannot be computed independently. Discovery of these problems led us to investigate the welfare
60 properties of the two systems by incorporating into a single theoretical model the two important
61 features noted above: branching rivers and nonlinear damages. Our purpose is to describe the
62 difficulties we encountered, to explain their source in each case, and to gauge the degree to which
63 they detract from the obvious appeal of the two schemes. Our ultimate finding is that both are
64 likely to serve reasonably well in practice. Attempts to apply them, though, should be carried out
65 with an awareness of the possible pitfalls.

66 After describing our model, we turn first to an analysis of Hung and Shaw’s trading-ratio system
67 (TRS). The TRS is designed to guarantee that ambient water-quality standards are satisfied at each
68 “zone” of the river system. The regulator first calculates a matrix of physical transfer coefficients,
69 a la Montgomery (1972), describing the portion of pollutant emitted at each zone that remains
70 in the river at any downstream zone. Permits are allocated so as to meet the zonal standards
71 without trade, and then trade between sources is executed at the ratio of coefficients. By design,
72 the transfer coefficient from a downstream to an upstream source, or across branches above a point
73 of confluence, is always zero. Thus, upstream sources cannot buy from downstream sources and
74 trade across branches above a point of confluence is prohibited. These restrictions mean that under
75 the TRS some trades that increase social welfare are prohibited. What is more, we find that when
76 a “hot spot” is located at a confluence of streams, the Hung-Shaw method of allocating permits
77 is ill defined. Where this is true, a cost-effective outcome can be guaranteed only if the regulator
78 knows firms’ abatement cost functions.

79 In Farrow *et al.*, whose methodology plays an important role in Muller and Mendelsohn (2009),
80 the main constraint is that total monetary damages in the river system cannot exceed a maxi-
81 mum determined by the regulator. (We call this the damage-denominated trading-ratio system,
82 or DTRS.) Each source is assigned a damage coefficient that reflects the integral over its down-
83 stream “zone of influence” of marginal damage caused by that source’s emissions. Permits are then
84 allocated so that aggregate damages satisfy the overall monetary constraint, and trade can occur
85 between any two sources at the ratio of their damage coefficients. Unlike the TRS, the DTRS is
86 not concerned with water quality at specific points along the river. Also unlike the TRS, trade can
87 occur across branches above a confluence and upstream sources can buy from downstream. When
88 the model is extended to nonlinear damages, though, we find that the Farrow *et al.* allocation
89 scheme too can be ill-defined. This possibility arises because emissions from each source affect the
90 marginal damages of all others. What is more, even if the damage coefficients are computed at the
91 cost-effective optimum, the trading outcome can still fail to achieve that optimum, because here
92 the initial supply of permits goes astray.

93 These findings suggest that we need to evaluate the welfare properties of the two systems
94 against the same benchmark. To this end, in Section 5 we consider the efficient, fully socially
95 optimal, program for a social planner who selects a vector of emissions to minimize the sum of
96 abatement costs and pollution damages. We compare the welfare losses of the two systems, if any,
97 against this efficient benchmark. We find that, when the regulator is able to allocate permits in the
98 first-best manner, the TRS struggles precisely where the DTRS succeeds (in the face of branching),
99 and vice versa (in the face of nonlinearities).

100 In particular, we find that the TRS outperforms the DTRS if there is no branching but damages
101 are nonlinear. Because the TRS sometimes disallows efficient trades, improvement over a no-trade
102 baseline can be inhibited. For certain configurations of sources above and below a confluence, in
103 particular if the disallowed trades would have been inefficient, this can turn out to be an advantage
104 relative to the DTRS. In contrast, however, when the regulator is unable to allocate permits in the
105 first-best manner, we find that the TRS is generally dominated by the DTRS. In such a case, the
106 regulator can make use of the damage coefficients to encourage efficient trades that increase welfare
107 relative to the second-best initial allocation of permits. The exception occurs when inefficient trades
108 between branches, prohibited by the TRS, are instead allowed by the DTRS. When this occurs,
109 the DTRS is too permissive in that some trades that reduce social welfare are permitted.

110 These findings suggest that the relative performance of the two systems depends on the dis-
111 tribution of sources in a watershed featuring both branching rivers and nonlinear damages. We
112 investigate this question by constructing a small numerical model and perturbing the geographic
113 distribution of pollution sources. Here again, our results suggest that neither system dominates
114 from a welfare perspective. As expected, the TRS precludes some efficient trades across branches
115 and the DTRS allows some inefficient trades. However, our comparative work tilts in favor of the
116 DTRS: for most of our limited set of configurations of the numerical model, welfare improvement
117 over the second-best allocation is larger for the DTRS than for the TRS. The question of how
118 the two compare in an empirical setting is deferred to future work. We find it encouraging that
119 the efficiency loss from failing to issue the correct number of permits is much greater than the
120 efficiency loss from failing to set the correct trading ratios. This last is what is meant by getting
121 prices right, and it leads us to believe the two systems do indeed represent significant innovations
122 in the water-quality trading literature.

123 The reader might reasonably ask whether hot spots at confluences, or nonlinear damages, are
124 important concerns in water policy. We believe that they are. Cities tend to rise up in those places
125 where rivers meet. This means that water quality is likely to be compromised by industrial activity,
126 and so a hot spot is more likely, at a confluence. Linear damages, on the other hand, certainly
127 confer a computational advantage upon a model of water-quality trading. But the chemistry and
128 the biology of aquatic systems are sometimes characterized by thresholds or tipping points or
129 other nonlinear relationships between nutrient concentration and outcomes of interest. What is
130 more, the constant marginal abatement benefits that attend linear damages are hardly standard
131 in environmental economics. In textbook treatments (Baumol and Oates 1988, for example) and a
132 host of scholarly articles, the assumption of declining marginal benefits is very much the norm.

2 A model of water-quality management

Consider the problem of regulating a single pollutant that is emitted by N point sources, indexed by $i = 1, \dots, N$, located along a river system. Let e_i represent emissions from source i , with $\mathbf{e} = (e_1, \dots, e_i, \dots, e_N)$, and let $\bar{\mathbf{e}}$ be a vector of baseline or uncontrolled emissions, with $e_i \leq \bar{e}_i$. Let $\mathbf{x} = (x_1, \dots, x_m, \dots, x_M)$ be a vector of ambient concentration levels, where x_m denotes concentration at receptor m . Assume, as in Montgomery (1972), that there exists a linear mapping $T : \mathcal{R}^N \rightarrow \mathcal{R}^M$ describing the scientific relationship between \mathbf{e} and \mathbf{x} , so that $\mathbf{x} = T\mathbf{e}'$, with T an $M \times N$ matrix of nonnegative transfer coefficients. The assumption of a linear mapping from emissions to concentration is not innocuous (Todd and Mays, 2005); we make it in order to place our focus firmly on nonlinear damages. Our framework would be useful also in understanding the implications of a nonlinear T mapping.

Let $S : \mathcal{R}^M \rightarrow \mathcal{R}$, given by $S(\mathbf{x})$, be a differentiable and possibly nonlinear function that describes total economic damages as a function of the vector of concentration levels and assume that $\partial S / \partial x_m > 0$ for all m . It follows that total economic damage as a function of emissions is differentiable and given by $D(\mathbf{e}) = S(T\mathbf{e}')$. Define a vector of abatement levels $\mathbf{a} = \bar{\mathbf{e}} - \mathbf{e}$, where by definition $a_i \in [0, \bar{e}_i]$. Each source i is assumed, here and throughout the paper, to have a twice-differentiable abatement cost function $C_i(a_i)$, with $C'_i > 0$, $C''_i > 0$, and $C_i(0) = 0$.

The usual approach to modeling an emissions-trading scheme is to specify one variant or another of a cost-effectiveness program. In Montgomery (1972), Krupnick *et al.* (1983), and McGartland and Oates (1985), for example, the constraint placed on the problem is a vector of environmental standards, one for each receptor. This approach is followed in Hung and Shaw (2005). For a given vector of exogenously determined zonal environmental standards $\bar{\mathbf{X}}$, they specify the following cost-effectiveness program:

$$\mathbf{a}^{\text{HS}} = \arg \min_{\mathbf{a}} \left\{ \sum_{i=1}^N C_i(a_i) \mid x_m \leq \bar{X}_m, \mathbf{x} = T\mathbf{e}', \text{ and } a_i \in [0, \bar{e}_i] \right\}. \quad (1)$$

The TRS is designed to solve program (1). We note that the TRS trading scheme is not designed to achieve a socially optimal outcome, a property it shares with earlier work.

An interesting alternative, the innovation of Farrow *et al.* (2005), is to specify a different cost-effectiveness program. Farrow *et al.* impose a constraint on total monetary damages caused by the vector of emissions. They assume that $D(\mathbf{e})$ is additively separable and linear in emissions: $D(\mathbf{e}) = \sum_i d_i e_i$, where d_i is a damage coefficient describing the aggregate damages caused by a unit of emissions from source i . For an exogenously given limit on total monetary damages $\overline{\text{TD}}$,

163 Farrow *et al.* specify the following cost-effectiveness program:

$$\mathbf{a}^{\text{FSCH}} = \arg \min_{\mathbf{a}} \left\{ \sum_{i=1}^N C_i(a_i) \mid D(\bar{\mathbf{e}} - \mathbf{a}) \leq \overline{\text{TD}} \text{ and } a_i \in [0, e_i] \right\}. \quad (2)$$

164 The DTRS is designed to solve program (2). Like the TRS, it is not designed to achieve a socially
 165 optimal outcome. As Muller and Mendelsohn (2009) observe, \mathbf{a}^{FSCH} will be socially optimal only
 166 if the constraint on total damages, $\overline{\text{TD}}$, is set at the efficient level.

167 Finally, we define a criterion that can be used to compare the two systems. Following Muller
 168 and Mendelsohn, a natural choice is to define the efficient program. A social planner who knows
 169 the cost and damage functions and who wishes to maximize social welfare will select an efficient
 170 vector of abatement that minimizes the sum of damages and abatement costs:

$$\mathbf{a}^{\text{eff}} = \arg \min_{\mathbf{a}} \left\{ \sum_{i=1}^N C_i(a_i) + D(\bar{\mathbf{e}} - \mathbf{a}) \mid a_i \in [0, e_i] \right\}. \quad (3)$$

171 Given that the C_i 's and D are continuous and that the constraint set is compact, the Weierstrass
 172 theorem ensures that a solution to (3) exists. Let \mathbf{x}^{eff} denote the associated vector of efficient
 173 concentrations and $D^{\text{eff}} = D(\bar{\mathbf{e}} - \mathbf{a}^{\text{eff}})$ the associated level of damages. In order to guarantee
 174 uniqueness of the solution to (3), one would need also to impose curvature restrictions on $D(\cdot)$. In
 175 the numerical simulation below, local curvature of a sigmoidal damage function yields optima that
 176 appear to be unique.

177 In the next two sections we show that in the presence of branching (manifest in the T matrix)
 178 the TRS equilibrium *may not achieve the solution to (1)*. If damages are nonlinear (manifest in the
 179 damage mapping S), the DTRS equilibrium *may not achieve the solution to (2)*. Therefore, one
 180 can guarantee neither that the equilibrium under the TRS is equivalent to that under the DTRS,
 181 nor that either system delivers the socially optimal vector of emissions. Whether these difficulties
 182 are significant in practice is an empirical question to which we turn in Section 6.

183 **3 The Trading-Ratio System (TRS)**

184 We begin by sketching the main elements of the TRS. For further details the reader should consult
 185 Hung and Shaw (2005). As in that paper, number the zones (and the sources) so that $i = 1$
 186 indicates the most upstream source and N the most downstream, where indexes on two branches
 187 above their confluence, though important for bookkeeping purposes, have no ordinal relationship
 188 to each other. (With one source per zone, the i and m indexes coincide.) Let the transfer matrix be
 189 $T = \{\tau_{ij}\}$, where τ_{ij} measures the water-quality impact of pollution from zone i upon concentration

190 at zone j . Given the unidirectional flow of a river, T has a special characteristic: for any m and n
 191 with $m > n$, or for zones on different branches, we must have that $\tau_{mn} = 0$. Following Hung and
 192 Shaw, we assume that each source influences its own zone in a unitary fashion: $\tau_{ii} = 1$ for all i .

193 Under the TRS, the allocation of tradable discharge permits begins at zone 1 and proceeds
 194 from there on down the stream, ensuring along the way that the concentration standard \bar{X}_i is met
 195 at each zone. This means that downstream sources may receive few permits or no permits in the
 196 initial allocation. This makes good economic sense, in that an efficient outcome should “fill the
 197 river” with pollution up to the standard at each receptor. Failing to do this will lead to higher
 198 aggregate abatement costs.

199 Given a vector $\bar{\mathbf{X}}$ of zonal concentration standards from (1), the TRS regulator uses the τ_{ij}
 200 to allocate zonal permits $\bar{\mathbf{Z}}$ so that the standards are met if no trade occurs. Define $\bar{Z}_1 = \bar{X}_1$
 201 and, for $j > 1$, define $\bar{Z}_j = \bar{X}_j - \sum_{i=1}^{j-1} \tau_{ij} \bar{Z}_i$. It is possible that, for a given j , we might find that
 202 $\tau_{(j-1)j} \bar{X}_{j-1} > \bar{X}_j$. That is, the level of pollution arriving from upstream when the standard is
 203 exactly met there exceeds zone j 's standard even if $e_j = 0$. In this case, zone j is called a *critical*
 204 *zone*. The TRS allocation scheme sets $\bar{Z}_j = 0$ and, in turn, reduces the allocation of permits to
 205 the upstream zone (or, possibly more than one upstream zone) to the point at which zone j is no
 206 longer critical: $\bar{Z}_{j-1} = (\bar{X}_j / \tau_{(j-1)j}) - \sum_{k=1}^{j-2} \tau_{kj-1} \bar{Z}_k$.

207 This allocation scheme ensures that the water-quality impacts of all upstream zonal standards
 208 on a given zone are accounted for via the upstream transfer coefficients. Note that in using the TRS
 209 procedure, the regulator takes as given the set of zones $\{i\}$, the zonal environmental standards $\bar{\mathbf{X}}$,
 210 and the transfer coefficients T . Each discharger is then allowed to trade freely in a watershed-wide
 211 permit market according to the transfer coefficients T , so long as its emissions do not exceed the
 212 permits it holds.

Formally, each source i solves:

$$\min_{r_{ki}, r_{si}, r_{sj}} C_i(a_i) - p_i r_{si} + \sum_j p_j r_{ji} \quad (4a)$$

$$\text{s.t. } \bar{Z}_i \geq (\bar{e}_i - r_{ki}) - \sum_{j=1}^{i-1} \tau_{ji} r_{ji} \quad (4b)$$

$$a_i = r_{ki} + r_{si} \quad (4c)$$

$$r_{si} = \sum_{j=i+1}^n r_{ij} \quad (4d)$$

$$r_{ki}, r_{si}, r_{sj} \geq 0, \quad (4e)$$

213 where p_i and p_j are the market prices of permits from sources i and j , r_{ji} is the amount of pollution
 214 control purchased from source j to offset pollution at source i , r_{ki} is the amount of pollution control

215 from source i that is kept by source i to meet the zonal standard \bar{Z}_i , and r_{si} is the amount of
 216 pollution control sold by source i . As Hung and Shaw observe, the TRS possesses two advantages
 217 over earlier trading schemes. The first is that each discharger must participate in only a single
 218 watershed-wide permit market, so that transaction costs are low. (Contrast this feature with the
 219 ambient-permit system of Montgomery (1972), where each polluter must hold permits for each of
 220 the receptor markets it affects.) The second is that the regulator allocates initial zonal discharge
 221 permits $\bar{\mathbf{Z}}$ in such a way that the ambient environmental constraints $\bar{\mathbf{X}}$ are satisfied exactly at the
 222 initial allocation.

223 One can rewrite (4b) to obtain Hung and Shaw's trading constraint (their equation (5)):

$$e_i \leq \bar{Z}_i + \sum_{j=1}^{i-1} \tau_{ji} r_{ji} - \sum_{j=i+1}^n r_{ij}, \quad (5)$$

224 where r_{ij} is the net amount of zonal discharge permits sold by source i to source j . This constraint
 225 means that any discharger can buy permits only from upstream zones and sell permits only to
 226 downstream zones. Because sources can trade permits at exchange rates τ_{ij} , in any TRS equilibrium
 227 (including the boundary case), for any $j > i$, the two permit prices must satisfy

$$\tau_{ij} p_j = p_i. \quad (6)$$

228 The economic implications of this equality are substantial. If a high-cost source is located
 229 upstream of, or on a different branch from, a low-cost source, because $\tau_{ij} = 0$ for $i > j$ this constraint
 230 strictly prohibits trade between them even if the trade would reduce costs. This might seem
 231 justifiable at first on the grounds that water flows downstream, so that any downstream pollution
 232 reduction or a reduction on a different branch has no effect on the concentration at the upstream
 233 location. However, if damages are nonlinear the downstream marginal damages of pollution from
 234 the high-cost (upstream) source can be comparable to those of the low-cost (downstream) source.
 235 In this case, increased abatement by the low-cost source in exchange for decreased abatement by the
 236 high-cost source might be Pareto improving. But this trade is infeasible under the TRS because of
 237 its prohibition on cross-branch or upstream sales, and so the TRS can fail to achieve the least-cost
 238 outcome. We shall return to this point in Section 6 when presenting the results of our numerical
 239 work.

240 We now turn to the main result of this section. The question is whether the TRS equilibrium
 241 is guaranteed to achieve the cost-effective outcome \mathbf{a}^{HS} . Proposition 1 shows that the answer is
 242 no. There are situations, perhaps not unusual in actual practice, in which the outcome of the TRS
 243 is either indeterminate (the permit-allocation scheme breaks down) or not cost effective (it fails to

244 solve program (1)). All proofs appear in the Appendix.

245 **Proposition 1.** *Consider a river system in which there exists a critical zone at the confluence of*
246 *upstream branches. Then the TRS permit-allocation scheme is indeterminate. The TRS equilibrium*
247 *is not guaranteed to achieve the cost-effective solution to program (1) unless the regulator knows*
248 *the cost functions of at least some upstream sources.*

249 Proposition 1 implies that the TRS cannot always be relied upon to deliver the cost-effective
250 outcome even if the ambient environmental constraints, the $\bar{\mathbf{X}}$, are set optimally. One might ask
251 whether the given condition, in which a critical zone lies at a confluence of branches, is likely to
252 be met in practice. We believe it is not at all unusual. In a branching river, confluence zone
253 m is critical if $\sum_{m-1_i} \tau_{(m-1_i)m} \bar{X}_{m-1_i} > \bar{X}_m$, where $\{m-1_i\}_i$ is the collection of indices directly
254 upstream of zone m , along all contributing branches. Economic activity and population both tend
255 to concentrate around the confluence of rivers. The water quality there is often important for both
256 aquatic species and people living nearby. Thus a zone of confluence might be more likely than
257 others to be critical.

258 4 The Damage-Denominated Trading-Ratio System (DTRS)

259 We turn now to an examination of the Farrow *et al.* DTRS. The fundamental regulatory constraint
260 in the DTRS is a single limit on aggregate monetary damages, here denoted $\overline{\text{TD}}$, rather than a set
261 of physical environmental standards. Hung and Shaw's TRS is deeply concerned over hot spots,
262 but, given the exogenous nature of the vector of standards, ignores damages. The DTRS, on the
263 other hand, is deeply concerned over damages but is concerned with hot spots only to the extent
264 that damages are high in some places along the river. The DTRS trading ratios are themselves
265 based upon marginal damages, rather than upon physical transfer coefficients. Each source i 's
266 marginal damage d_i is calculated by integrating its contribution to monetary damages over that
267 source's "zone of influence." Having calculated marginal damages for each source, the regulator
268 distributes permits \bar{L}_i (in terms of emissions at the point of discharge) in such a way that aggregate
269 damages meet the overall monetary constraint at the initial allocation: $\sum_i d_i \bar{L}_i = \overline{\text{TD}}$. Trade is
270 allowed between any two sources, but at the ratio of their marginal damages. The aggregate limit
271 on damages will be satisfied in the face of any permissible trade at these ratios.

272 Given the vector \mathbf{d} of marginal damages and a vector \mathbf{e} of emissions, Farrow *et al.* (and also
273 Muller and Mendelsohn 2009) assume that aggregate damages are linear: $D(\mathbf{e}) = \sum_{i=1}^n d_i e_i$. It is
274 this quantity that must not exceed $\overline{\text{TD}}$. The assumed linearity of the damage function means that
275 each d_i is independent of emissions from other sources.

276 Each source i solves the following cost-minimization program:

$$\min_{r_{ki}, r_{si}, r_{sj}} C_i(a_i) - p_i r_{si} + \sum_j p_j r_{ji} \quad (7a)$$

$$\text{s.t. } (\bar{e}_i - r_{ki}) - \sum_j \frac{d_j}{d_i} r_{ji} \leq \bar{L}_i \quad (7b)$$

$$a_i = r_{ki} + r_{si} \quad (7c)$$

$$r_{ki}, r_{si}, r_{sj} \geq 0, \quad (7d)$$

277 where p_i and p_j are the market prices for permits from sources i and j , r_{ji} is the amount of pollution
 278 control purchased from source j to offset pollution at source i , r_{ki} is the amount of pollution control
 279 from source i that is kept by that source to meet its emissions standard \bar{L}_i , and r_{si} is the amount
 280 of pollution control sold by source i .

281 Note that substituting $e_i = \bar{e}_i - a_i$ and $r_{ki} = a_i - r_{si}$ into (7b), one obtains an analogue of (5),
 282 the Hung-Shaw trading constraint:

$$e_i \leq \bar{L}_i + \sum_j \frac{d_j}{d_i} r_{ji} - r_{si}.$$

283 This constraint means that each source can trade with any other source, according to the marginal
 284 damage ratios, so long as the level of its discharge does not exceed the sum of the original discharge
 285 limits \bar{L}_i and the net purchase of damage-denominated permits $\sum_j (d_j/d_i) r_{sj} - r_{si}$. Because sources
 286 can trade permits at the exchange rates d_j/d_i , the spatially explicit prices of permits in equilibrium
 287 (including the boundary case) satisfy the following analogue of (6):

$$\frac{d_j}{d_i} p_j = p_i. \quad (8)$$

288 Note that unlike in the TRS, one can be sure that $d_i \neq 0$ in practice for all i : a source for which
 289 $d_i = 0$ would not be part of the trading system. Therefore, each source can trade with any other
 290 source, including those located upstream or downstream or on different branches of the river.

291 Farrow *et al.* derive the first-order necessary (and sufficient) conditions for each source's opti-
 292 mization problem, from which the following *interior* equilibrium condition is derived:

$$\frac{C'_i(a_i)}{C'_j(a_j)} = \frac{d_i}{d_j} = \frac{p_i}{p_j}. \quad (9)$$

293 In fact, in equilibrium the right equality in (9) (and therefore also (8)) must be satisfied for every
 294 pair i and j , not just those who actually trade. Unlimited arbitrage opportunities would arise if it
 295 were violated for any pair. To see this, note that if (8) does not hold, say, if $p_i/d_i > p_j/d_j$, then
 296 any source k who buys permits from j and sells them to i can secure such profits. But this cannot

297 be an equilibrium with a finite supply of permits.

298 Take as given a vector of baseline emissions $\bar{\mathbf{e}}$, a vector of initial permits $\bar{\mathbf{L}}$, and a vector
 299 of trading ratios \mathbf{d} . The complete characterization of the Farrow *et al.* equilibrium conditions is
 300 given in (10a)–(11c). These expressions, which we use in our numerical example, are stated for
 301 completeness. Thus, we provide only a sketch of their derivation. The first set, equations (10a)–
 302 (10c), characterize an *interior* equilibrium. Letting $R_i(p_i)$ denote i 's abatement decision function
 303 given p_i , we have

$$a_i^* = R_i(p_i^*), \quad (10a)$$

$$\sum_i d_i [\bar{e}_i - R_i(p_i^*) - \bar{L}_i] = 0, \quad (10b)$$

$$\frac{p_i^*}{d_i} = \frac{p_j^*}{d_j} \quad \text{for all } i, j. \quad (10c)$$

304 Equation (10c), which like the right equality in (9) serves as a no-arbitrage condition, ensures that
 305 each source i faces the same effective price in all zonal markets j : $p_i = p_j(d_i/d_j)$. Source i is
 306 therefore indifferent from which sources it buys permits or to which sources it sells. It follows that
 307 source i abates such that $C'_i(a_i) = p_i$. Because $C'_i(a_i)$ is strictly increasing, the interior optimal
 308 abatement a_i^* is unique. Thus $p_i = p_j(d_i/d_j)$ for all j .

309 Equations (11a)–(11c) characterize the vector of equilibrium traded quantities, $\{r_{ki}^*, r_{si}^*, \sum_j \frac{d_j}{d_i} r_{ji}^*\}_{i,j}$,
 310 while accounting for the relevant corners:

$$r_{ki}^* = \begin{cases} \bar{L}_i & \text{if } \bar{e}_i - R_i(p_i^*) \leq \bar{L}_i \\ 0 & \text{if } \bar{e}_i - R_i(p_i^*) > \bar{L}_i \end{cases} \quad (11a)$$

$$r_{si}^* = \begin{cases} \bar{L}_i - \bar{e}_i + R_i(p_i^*) & \text{if } \bar{e}_i - R_i(p_i^*) \leq \bar{L}_i \\ 0 & \text{if } \bar{e}_i - R_i(p_i^*) > \bar{L}_i \end{cases} \quad (11b)$$

$$\sum_j \frac{d_j}{d_i} r_{ji}^* = \begin{cases} 0 & \text{if } \bar{e}_i - R_i(p_i^*) \leq \bar{L}_i \\ \bar{e}_i - R_i(p_i^*) - \bar{L}_i & \text{if } \bar{e}_i - R_i(p_i^*) > \bar{L}_i \end{cases} \quad (11c)$$

311 Source i 's excess demand function may be obtained as follows. Given p_i , source i would choose
 312 abatement R_i so that $C'_i = p_i$. Thus, R_i is a well-defined function. The excess demand for permits
 313 from source i is $z_i(p_i; \bar{e}_i, \bar{L}_i) = \bar{e}_i - R_i(p_i) - \bar{L}_i$. If $z_i > 0$, then i must buy permits. If $z_i < 0$, it sells
 314 its excess permits. All permits sold to and purchased from i must be exchanged at the ratio d_i/d_j
 315 with permits from any source j . This means that the common units of exchange are $d_i z_i$, and so
 316 the market clears in equilibrium if equations (10a)–(11c) hold. For a given vector $\{\bar{e}_i, \bar{L}_i, d_i\}_i$ and

317 n sources, this gives us n equations in n unknown prices $\{p_i^*\}_i$. Thus the equilibrium is exactly
 318 identified.

319 This characterization of market equilibrium turns out to be useful for the simulations in Section
 320 6. There may be non-trivial boundary equilibria in which $a_i^* = 0$ or $a_i^* = \bar{e}_i$. These boundary cases
 321 can be dealt with by defining $R_i(p_i^*) = 0$ if $C'_i(a_i) > p_i^*$ for all $a_i \in [0, \bar{e}_i]$ and $R_i(p_i^*) = \bar{e}_i$ if
 322 $C'_i(a_i) < p_i^*$ for all $a_i \in [0, \bar{e}_i]$. The rest of the equilibrium conditions are intact.

323 Our next result is analogous to Proposition 1. The question is whether the DTRS equilibrium
 324 is guaranteed to achieve the solution to program (2) if damages are nonlinear. Proposition 2 shows
 325 that the answer is no.

326 **Proposition 2.** *Suppose that aggregate environmental damages are a nonlinear function of pollu-*
 327 *tion concentration, so that at the cost-effective solution \mathbf{e}^{FSCH} we have*

$$D(\mathbf{e}^{FSCH}) \neq \sum_i \frac{\partial D(\mathbf{e}^{FSCH})}{\partial e_i} e_i^{FSCH}.$$

328 *Then the DTRS equilibrium does not achieve the cost-effective solution to program (2), even if the*
 329 *d_i are evaluated at the optimum.*

330 Because Farrow *et al.*'s system assumes linear damages, the result that the DTRS breaks down
 331 in the face of nonlinear damages is perhaps not surprising, though it has not evidently been noted
 332 before. More surprising is that with nonlinear damages the DTRS fails to achieve the cost-effective
 333 solution even if the regulator evaluates the trading ratios at the efficient allocation. The DTRS
 334 equilibrium can get not only the price ratios, but also the aggregate market-clearing condition,
 335 wrong. Whether this is significant in practice is an empirical question.

336 As is evident from the proof, the difficulty with the DTRS stems from the fact that the initial
 337 allocation of permits follows Farrow *et al.*'s original allocation rule, (18). A natural question arises:
 338 what would happen if one were to use a different allocation rule? For example, the regulator could
 339 allocate permits so that $D(\bar{L}_1, \dots, \bar{L}_n) = \overline{\text{TD}}$. Here one encounters an insuperable difficulty: there
 340 is no allocation rule the regulator could rely upon in this case. Indeed, the problem is similar to
 341 that of the TRS. To see this, suppose that the regulator agreed upon the desired level of aggregate
 342 damage, $\overline{\text{TD}}$. Because the damage function is nonlinear, there will inevitably exist many vectors
 343 $\bar{\mathbf{L}}$ such that $D(\bar{L}_1, \dots, \bar{L}_n) = \overline{\text{TD}}$. The regulator's problem is indeterminate. (Recall that D is the
 344 composition function $D(\mathbf{e}) = S(T\mathbf{e})$.)

345 In the following section, we investigate how the TRS and DTRS perform, relative to the efficient
 346 solution as well as to each other, if the initial allocation of permits follows such a rule.

347 **5 Equilibrium comparisons**

348 Given the performance of the TRS and the DTRS, respectively, in the face of branching and
 349 nonlinear damages, a natural question is whether it is possible to say which of the two is preferred,
 350 based either on theoretical arguments or on empirical evidence. In this section we explore this
 351 question theoretically, always bearing in mind that empirical considerations in any given situation
 352 will likely be decisive.

353 The comparative question really does matter, for at least these two reasons. One is that, as we
 354 have shown, the initial allocation of zonal standards under the TRS is indeterminate in the presence
 355 of branching. The initial allocation of permits under the DTRS is indeterminate in the presence of
 356 nonlinear damages. One would like to know whether either of these difficulties is cause for concern
 357 and, if so, which is the greater. The other is that the informational requirements in applying the
 358 two schemes might appear to be different: the DTRS incorporates information on damages while
 359 the TRS does not. We will see that, in fact, the information needed to deploy either scheme is the
 360 same.

361 Proposition 3 shows how the TRS and the DTRS can both be derived directly from the efficient
 362 program found in (3). Thus, the two alternatives can be compared on the same efficiency grounds
 363 within our framework.

364 **Proposition 3.** *Under the conditions imposed upon the C_i and D , the following are true:*

- 365 (i) *Given the efficient solution \mathbf{a}^{eff} , there exists a constraint vector $\bar{\mathbf{X}}^{eff}$ in terms of pollution*
 366 *concentrations such that the solution \mathbf{a}^{HS} to program (1) subject to $\bar{\mathbf{X}}^{eff}$ is the optimal solu-*
 367 *tion \mathbf{a}^{eff} ; and*
- 368 (ii) *Given the efficient solution \mathbf{a}^{eff} , there exists a constraint value \overline{TD}^{eff} in terms of total dam-*
 369 *ages such that the solution \mathbf{a}^{FSCH} to program (2) subject to \overline{TD}^{eff} is the optimal solution*
 370 *\mathbf{a}^{eff} .*

371 Proposition 3 establishes the practical equivalence of the two cost-effective programs in a situ-
 372 ation in which the regulator has perfect knowledge of T and D . (Note that in order to achieve the
 373 social optimum the by either system, the regulator must also know the $C_i(a_i)$.) In that case it does
 374 not matter whether the policy is constrained by $\bar{\mathbf{X}}$ or by \overline{TD} . Suppose, though, that the regulator
 375 has imperfect information on one of these elements. Even if there is no branching and damages
 376 are linear, the informational requirements of the two trading mechanisms are quite different. A
 377 regulator wishing to implement the TRS must estimate the transfer coefficients in T while a regu-
 378 lator wishing to implement the DTRS must estimate the damage coefficients, the d_i . Thus for our

379 comparative work, we must define the requisite policy choice for a regulator who wishes to achieve
 380 \mathbf{e}^{eff} in each system. Under the TRS, the regulator must select zonal environmental standards at
 381 the efficient levels: $\bar{\mathbf{X}}^{FB} = T\mathbf{e}^{\text{eff}}$. Under the DTRS, the regulator must constrain total damages at
 382 the efficient level: $\overline{\text{TD}}^{FB} = D(\mathbf{e}^{\text{eff}})$. Let us call any distribution of permits consistent with $\bar{\mathbf{X}}^{FB}$
 383 under the TRS, or $\overline{\text{TD}}^{FB}$ under the DTRS, the “first-best” allocation.

384 Given our Propoposition 3, we shall employ the following criterion for judging which of the two
 385 schemes outperforms the other.

386 **Definition.** *Given two feasible abatement vectors \mathbf{a} and \mathbf{a}' , say that \mathbf{a} is at least as efficient as*
 387 *\mathbf{a}' if $\sum_i C_i(a_i) + D(\bar{\mathbf{e}} - \mathbf{a}) \leq \sum_i C_i(a'_i) + D(\bar{\mathbf{e}} - \mathbf{a}')$.*

388 To prepare for our final results, we first establish that the TRS equilibrium achieves the efficient
 389 outcome, if it achieves it at all, *with no trade*. To see this, consider a branchless river system. Note
 390 that as in the proof of Proposition 3, the efficient environmental constraints are found by setting
 391 $\bar{\mathbf{X}}^{\text{eff}} = T(\bar{\mathbf{e}} - \mathbf{a}^{\text{eff}})$. Then as Hung and Shaw show, in a branchless river the constraint set arising
 392 from $\bar{\mathbf{Z}}$ is equivalent to that arising from $\bar{\mathbf{X}}^{\text{eff}}$ and the TRS equilibrium achieves the cost-effective
 393 outcome. But because the cost-effective outcome must coincide with the efficient outcome, which
 394 also coincides with the initial allocation, and because it is assumed that there is only one discharger
 395 in each zone, this implies that discharging pollution so as to satisfy $\bar{\mathbf{Z}}$ exactly, without engaging in
 396 any trade, is also cost-minimizing. Put another way, the regulator cannot implement the efficient
 397 optimum in a decentralized manner. This claim, whose proof is obvious and so is omitted, is stated
 398 in the following result.

399 **Proposition 4.** *Suppose that in program (1), the zonal environmental constraints $\bar{\mathbf{X}}$ are set at the*
 400 *efficient levels and that there is only one discharger in each zone. Then if the TRS trading achieves*
 401 *the cost-effective optimum of program (1), it is achieved with no trade.*

402 The next result follows in a straightforward manner from Propositions 1, 2, and 3, and so the proof
 403 is omitted.

404 **Proposition 5.** *Suppose the regulator allocates permits in the first-best manner. Then the following*
 405 *are true:*

- 406 (i) *If the watershed is characterized by a branchless river but a nonlinear damage function S , the*
 407 *equilibrium under the TRS scheme is at least as efficient as that under the DTRS scheme;*
- 408 (ii) *If the watershed is characterized by a branching river but a linear damage function S , the*
 409 *equilibrium under the DTRS scheme is at least as efficient as that under the TRS scheme.*

410 This result may come as a surprise. Why can the DTRS not outperform the TRS when en-
411 vironmental damages are nonlinear, even though the DTRS incorporates some information about
412 variation in marginal damages whereas the TRS does not? The answer is precisely that when
413 the regulatory benchmark is the efficient outcome, the regulator *must* commit herself to an initial
414 allocation of permits that meets the constraints $\bar{\mathbf{X}}^{FB}$ (for the TRS) or $\overline{\text{TD}}^{FB}$ (for the DTRS). The
415 difficulty with the DTRS when damages are nonlinear is with the dependence of the d_i 's upon the
416 entire vector of emissions. The price signals offered by the physical transfer coefficients under the
417 TRS, which are independent of abatement decisions, are consistent with the first-best outcome.
418 The price signals offered by the variable damage coefficients, the d'_i , are not.

419 In practice, even a perfectly informed regulator might face informational or political constraints
420 that drive the allocation of permits away from the first-best allocation. Selecting a useful criterion
421 for comparison becomes more difficult in this case. The reason is that, as may be seen in (20),
422 when damages are nonlinear it can be the case that the realized damages at a DTRS equilibrium
423 actually exceed the constraint value of $\overline{\text{TD}}$ that generated the permit allocation. Thus, just how
424 one chooses the appropriate “second-best” standards, $\bar{\mathbf{X}}^{SB}$ or $\overline{\text{TD}}^{SB}$, so as to allow a legitimate
425 comparison between the TRS outcome and the DTRS outcome is not at all obvious. The TRS
426 equilibrium is sure to produce the desired level of damages, but the DTRS equilibrium is not.

427 A reasonable alternative might be to compare equilibria *given an initial allocation of permits*.
428 Assuming that there is one discharger in each zone, and that $\tau_{ii} = 1$ for all i as in Section 3,
429 specifying the initial allocation in terms of zonal standards $\bar{\mathbf{X}}$ under the TRS is equivalent to
430 specifying the initial allocation $\bar{\mathbf{L}}$ under the DTRS. Permits would then be allocated at $\bar{\mathbf{X}}^{SB} = \bar{\mathbf{L}}^{SB}$
431 so that

$$S(\bar{\mathbf{X}}^{SB}) = D(\bar{\mathbf{L}}^{SB}) \neq D(\mathbf{e}^{\text{eff}}).$$

432 In this case, we would be comparing the performance of the two trading systems relative to a
433 no-trading baseline. The following result offers some evidence that the comparison leans more in
434 favor of the DTRS than the TRS.

435 **Proposition 6.** *Suppose the regulator allocates permits in a second-best manner. Then the follow-*
436 *ing are true:*

- 437 (i) *If the watershed is characterized by a branchless river but a nonlinear damage function S , the*
438 *equilibrium under the DTRS scheme can be more efficient than that under the TRS scheme;*
439 *and*
- 440 (ii) *If the watershed is characterized by a branching river but a linear damage function S , the*
441 *equilibrium under the DTRS scheme is no less efficient than that under the TRS scheme.*

442 A caveat is in order. One may be tempted to interpret this result as saying that the DTRS
443 dominates the TRS under this second-best condition. However, that interpretation could be mis-
444 leading. First, it seems implausible that the regulator would be able to choose d_i 's with sufficient
445 accuracy to ensure that equations (15) are satisfied. Second, many river systems are likely to be
446 characterized by *both* branching and nonlinear damages. We need to examine the relative perfor-
447 mance of the two systems when both properties are present. The following section uses a numerical
448 model to explore this question.

449 6 A numerical model

450 In this section we develop and solve a small numerical model aimed at providing an answer to this
451 question: which of the systems, TRS or DTRS, performs better, relative to each other and to the
452 efficient outcome, when river systems are characterized by both branching and nonlinear damages?
453 Our answer comes in two forms. One is that neither system dominates, and which performs better
454 depends on the distribution of firms' abatement costs. The other is that even in the presence of
455 both branching and nonlinear damages the TRS and the DTRS perform well so long as the total
456 quantity of permits issued is close to the optimal level. Getting prices right (that is, setting the
457 correct trading ratios) is much less important than getting the quantity right.

458 For purposes of this numerical exercise, we suppose that a regulator knows the damages caused
459 by water pollution. She also knows enough of the science to be able to specify the transfer coefficients
460 correctly. The regulator is imperfectly informed about abatement cost functions, however, and so
461 cannot compute the socially optimal vector of emissions. Put another way, our regulator is unable
462 to set either $\bar{\mathbf{X}}$ (for the TRS) or $\bar{\mathbf{T}}\bar{\mathbf{D}}$ (for the DTRS) at the efficient levels. We start out, though,
463 by giving our regulator a helpful nudge. That is, we set the total number of permits equal to the
464 socially optimal level. Any divergence between the social optimum and the outcomes of the TRS
465 or the DTRS, then, cannot be due to getting the quantity of permits wrong. It must be because
466 the trading schemes provide incorrect price incentives. At the end of the section we consider the
467 added inefficiency that results from an incorrect quantity of permits.

468 The details of our numerical model are adapted from the EPA's (2002) tool for modeling water-
469 quality impacts, the NWPCAM, which is also the basis for the empirical application in Farrow *et al.*
470 (2005). Let x_{mi} be source i 's contribution to the pollution concentration at location m downstream.
471 Then x_{mi} is a function of the emissions e_i at source i , stream flow Q , and an exponential decay
472 term:

$$x_{mi} = \frac{e_i}{Q} \exp\left(-\hat{k}\delta_{mi}\right), \quad (12)$$

473 with $x_{mi} = 0$ if m is upstream of i . In (12), which simplifies the Farrow *et al.* version slightly, \hat{k} is
 474 the decay parameter and δ_{mi} is distance in river miles. The pollution concentration at location m
 475 is the sum of contributions from all upstream sources: $x_m = \sum_i x_{mi}$. This model can incorporate
 476 branching in the river. If two sources, i and j , are located along two different branches upstream
 477 of a confluence, the impacts of emissions from i and j on concentrations at location k below the
 478 confluence are simply x_{ki} and x_{kj} . In this framework the effect of changes in concentration at
 479 location i on concentration at any downstream location j is linear:

$$\tau_{ij} \stackrel{def}{=} \frac{dx_j}{dx_i} = \exp(-\hat{k}\delta_{ij}), \quad (13)$$

480 where the τ_{ij} are the transfer coefficients.

481 In order for our model to capture the effects of interest, it must be able to accommodate both
 482 nonlinear damages and critical zones where branching occurs. The latter is straightforward, but
 483 the specific form of nonlinearity is crucial and requires a bit of explanation. Consider first the
 484 linear specification of damages found in Farrow *et al.*, who assumed that marginal damages are
 485 constant at each location: $\partial D/\partial x_m = \text{WTP} \times H_m$, where WTP is the constant per-capita marginal
 486 damages from changes in water quality and H_m is the population at location m . According to this
 487 specification, damages from each source's emissions are given by $D_i(e_i) = d_i e_i$, where d_i is constant
 488 and independent of e_i :

$$d_i = \sum_{m=1}^M \text{WTP} \times H_m \times \tau_{mi} \times \frac{1}{Q}.$$

489 To justify the constant marginal willingness to pay (WTP), Farrow *et al.* (2005, p. 197) argue that
 490 water quality is inversely related to pollution concentrations and that “the household marginal
 491 willingness to pay for a small improvement in water quality, WTP, is constant . . . over the range of
 492 water quality conditions considered in this study.”

493 Linearity may often be a reasonable assumption, but we argue that there are important cases
 494 where it is not. One source of nonlinearity is curvature in the biology or chemistry of aquatic
 495 systems. The relevant science, including that for fish and for algae, indicates that the underlying
 496 biology can be nonlinear as a function of water quality (see Elliott 2000; Schnoor 1996; and Anderson
 497 *et al.* 2002). In such cases, economic damages may be expected to be nonlinear too.

498 The effect of thresholds can also be important. In many watersheds in the U.S., water quality
 499 is already impaired, so that a further increase in pollution concentrations might drive conditions
 500 above a particular concentration threshold. Indeed, our communications with water practitioners
 501 and policymakers reveal that their primary concern is often related to this nonlinear biological
 502 response around hotspots. This concern is one of the important political factors that has plagued

	Parameter	Units	Value
503	k Decay rate	mile ⁻¹	0.005
504	Q Stream flow	ft ³ /s	10
505	a Damage parameter	none	5
506	b Damage scale parameter	none	6.7
507	c Concentration threshold	mg/L	5

Table 1: Parameters for water-quality model

503 water-quality trading in many watersheds. Understanding how a trading scheme performs in the
504 face of nonlinearity, we argue, is essential.

505 To model the nonlinear biological responses that allow for the type of threshold effects high-
506 lighted above, we consider a logistic damage response to pollution concentrations at each location
507 m :

$$S_m(x_m) = \frac{b}{1 + \exp(-a(x_m - c))} \quad \text{for all } m = 1, \dots, M, \quad (14)$$

508 where a is a damage-sensitivity parameter, b a scale parameter, and c a concentration threshold.
509 The total economic damages are then given by $D(\mathbf{e}) = \sum_m S_m(x_m)$. Logistic models are commonly
510 used in biology and ecology for modelling the response of species' mortality or population size to
511 pollution. Though in biology, the parameters a and c often depend on a variety of environmental
512 factors, for simplicity of analysis we treat them as constants that do not vary by time or location.
513 The parameter b is a scaling parameter that transforms biological damages into monetary economic
514 damages, which we also assume are constant. With these assumptions, marginal aggregate damages
515 with respect to emissions from any source i depend not only upon emissions from that source but
516 also upon emissions from other sources, both downstream and upstream of i (that is, $\partial D / \partial e_i =$
517 $\sum_m (\partial S_m / \partial x_m) (\partial x_m / \partial e_i)$). When there is a branch in a river system, marginal damages also
518 depend on the emissions from sources located along the other branches.

519 The parameter values of the model can vary widely by pollutant and watershed. Because
520 our goal is to obtain generic efficiency properties of the two trading systems, we decided to choose
521 representative parameter values for the water-quality model (12) and then choose a set of parameters
522 a , b , and c that generate interior optima for at least two out of three sources given the assumed
523 cost parameters (see below). The value of $\hat{k} = 0.005$ is chosen based on three parameters: the
524 mean of the decay rates for seven representative water pollutants (U.S. EPA, 2002), the average
525 water temperature of 20 Celsius degrees, and the stream velocity of 1.5 miles per hour. The scale
526 parameter b and the threshold parameter c are important in generating interior solutions. We thus

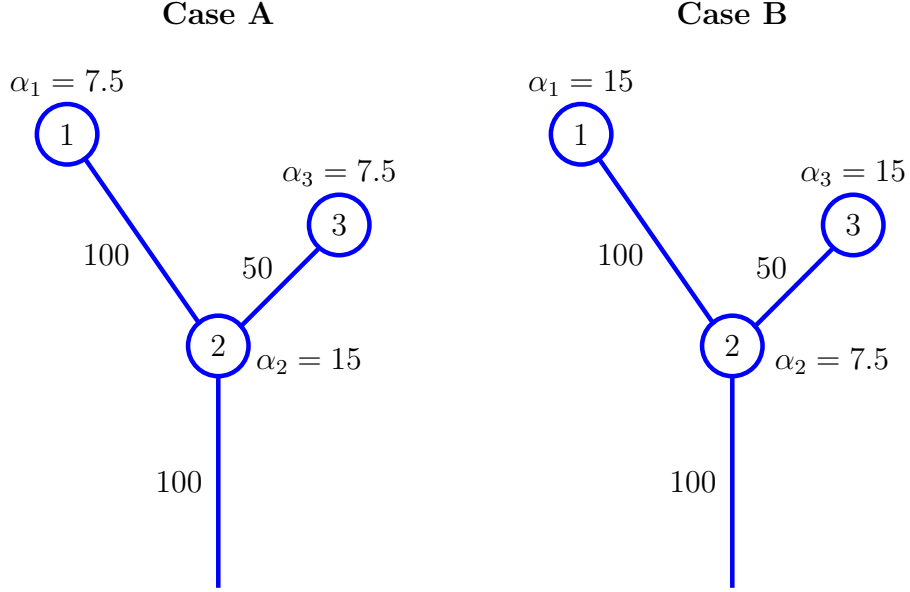


Figure 1: Hypothetical river system.

527 started with arbitrary values $a = 5$ and $c = 5$ and then searched for the associated value of b . The
 528 parameters used for the simulation are summarized in Table 1.

529 6.1 Simulation scenarios

530 We assume that the river has a main stem M and a single branch B . The river has a maximum
 531 length of 200 river miles along the main stem ($m^M \in [0, 200]$) and 50 river miles along the branch
 532 ($m^B \in [0, 50]$) above a confluence at $m^M = 100$ (or at $m^B = 50$). There are three polluting
 533 sources. Source 1 is located in the most upstream point of the main stem ($m^M = 0$), source 2 at
 534 the confluence ($m^M = 100$ or $m^B = 50$), and source 3 at the most upstream point of the branch
 535 (see Figure 1). There is one pollution source i in each zone i , as in Hung and Shaw (2005). Thus
 536 there are three zones, each zone facing zonal environmental standard \bar{X}_i and receiving zonal permits
 537 $\bar{Z}_i = \bar{L}_i$. The firms have quadratic abatement cost functions of the form

$$C_i(a_i) = \frac{a_i^2}{\alpha_i}, \quad (15)$$

538 with $a_i \in [0, \bar{e}_i]$ and $\alpha_i > 0$.

539 Given the transfer coefficients as in (13), the damage function in (14), and the cost functions
 540 in (15), we first compute the socially optimal vector of emissions, \mathbf{e}^{eff} . Again, our regulator does
 541 not know cost functions, and so cannot solve for this optimum. A fair comparison of outcomes

542 of the TRS and the DTRS to the optimum, though, requires that we set the total number of
 543 permits at the optimal level. The optimal total is then allocated equally across the three sources:
 544 $\bar{L}_1 = \bar{L}_2 = \bar{L}_3 = \sum e_i^{\text{eff}}/3$. This choice of an initial allocation is only one of an infinite number of
 545 possibilities, but it is a reasonable choice for a regulator who does not know costs.

546 Under the TRS, this means that zonal environmental standards, the \bar{X} 's, are allocated so that

$$\bar{X}_1 = \bar{L}_1, \quad \bar{X}_3 = \bar{L}_3, \quad \text{and} \quad \bar{X}_2 = \bar{L}_2 + \tau_{12}\bar{X}_1 + \tau_{32}\bar{X}_3.$$

547 We chose this allocation rule for two reasons. The first is that we are interested in the relative
 548 performance of the two trading systems under conditions that can be compared to the social
 549 optimum. The second is that, were we instead to allocate permits so as to ensure that water
 550 quality is equal in all zones, by design we would have a critical zone at confluence zone 2. This
 551 would mean in turn that the problem of indeterminate allocation would arise (see Proposition 1).

552 Under the TRS, the correct transfer coefficients are known to the regulator and are announced
 553 to the polluters. Under the DTRS, the regulator does not know the social optimum, and so she
 554 evaluates the d_i 's at the initial allocation. In each of these setups, we simulate the trading outcomes
 555 for two sets of cost parameters:

$$\text{Case A: } \alpha_1 = 7.5, \quad \alpha_2 = 15, \quad \alpha_3 = 7.5 \quad \text{and}$$

$$\text{Case B: } \alpha_1 = 15.0, \quad \alpha_2 = 7.5, \quad \alpha_3 = 15.0.$$

556 For each case, we also compute the social costs at the initial allocation of permits, as the no-trading
 557 baseline. These two cases represent only a fraction of the infinity of possible arrangements, but
 558 they do provide some useful insights.

559 6.2 Simulation results

560 **Case A.** In this case, a low-cost firm is located downstream of two high-cost firms. At the socially
 561 optimal outcome, the low-cost firm (source 2) abates completely while source 1 emits more than
 562 source 3 even though they have the same marginal costs and the same baseline emissions (Table 2).
 563 This occurs because marginal damages at the optimum increase more in source 3's emissions than
 564 in source 1's emissions. Recall that the TRS mechanism precludes upstream sales and trade across
 565 branches. Because the potential seller (the low-cost firm 2) is located downstream, that source
 566 cannot trade at all. Moreover, the socially optimal trade between the two upstream firms is also
 567 precluded. As a result, firms incur higher abatement costs under the TRS than at the optimal

Case A	Outcome	e_1	e_2	e_3	Damage	Cost	Total
	No trade	23.7	23.7	23.7	511	1,942	2,454
	TRS	23.7	23.7	23.7	511	1,942	2,454
	DTRS	48.9	0.0	34.4	530	1,589	2,119
	Optimum	42.0	0.0	29.0	60	1,787	1,848
Case B	Outcome	e_1	e_2	e_3	Damage	Cost	Total
	No trade	18.5	18.5	18.5	10	1,771	1,782
	TRS	18.5	32.0	0.0	10	1,710	1,720
	DTRS	20.5	31.4	0.0	9	1,716	1,725
	Optimum	21.0	34.5	0.0	42	1,655	1,697

Table 2: Simulation results

568 outcome. On the other hand, the DTRS does allow trades among any of the three sources. Because
569 of this, the DTRS performs substantially better than the TRS (and therefore, the no-trading
570 baseline).

571 A difficulty with the DTRS, however, is that in our simulation the d_i do not appear to be good
572 approximations to the assumed marginal damages at the optimum. This point is demonstrated
573 in Figure 2, which plots marginal damages as a function of each source's emissions, holding other
574 sources' emissions at the optimum. Figure 2 also shows each source's marginal cost and trading
575 coefficient. The social optimum occurs where each firm's marginal damages are equated with its
576 marginal cost and the overall constraint is satisfied. Interestingly, the equilibrium does not occur
577 where each source's marginal cost equals its trading coefficient d_i . This is because each source makes
578 its abatement and trading decisions so that its marginal cost equals the spatially explicit price it
579 faces, $p_i = (d_j/d_i)p_j$. Thus, when the d_i 's do not closely approximate actual marginal damages, the
580 trading outcome under DTRS may not equate marginal damages with marginal costs. In actual
581 practice, of course, it might be true that damages are nearly linear, so that the DTRS is more
582 nearly optimal than in our example.

583 Note also that in the DTRS equilibrium the sum of emissions can exceed the sum of initial
584 emissions permits: $\sum e_i^{\text{DTRS}} > \sum e_i^{\text{eff}}$. This is because neither the individual polluters nor the
585 equilibrium market-clearing condition are constrained by $\sum e_i^{\text{DTRS}} \leq \sum e_i^{\text{eff}}$. In the DTRS, the
586 common unit of exchange is $d_i e_i$ rather than e_i itself. Thus, the discipline imposed on the equilib-
587 rium is that $\sum d_i e_i^{\text{DTRS}} \leq \sum d_i \bar{L}_i$. Because sources can emit more than the socially optimal total,
588 environmental damages are higher, but abatement costs are lower, at the DTRS equilibrium than

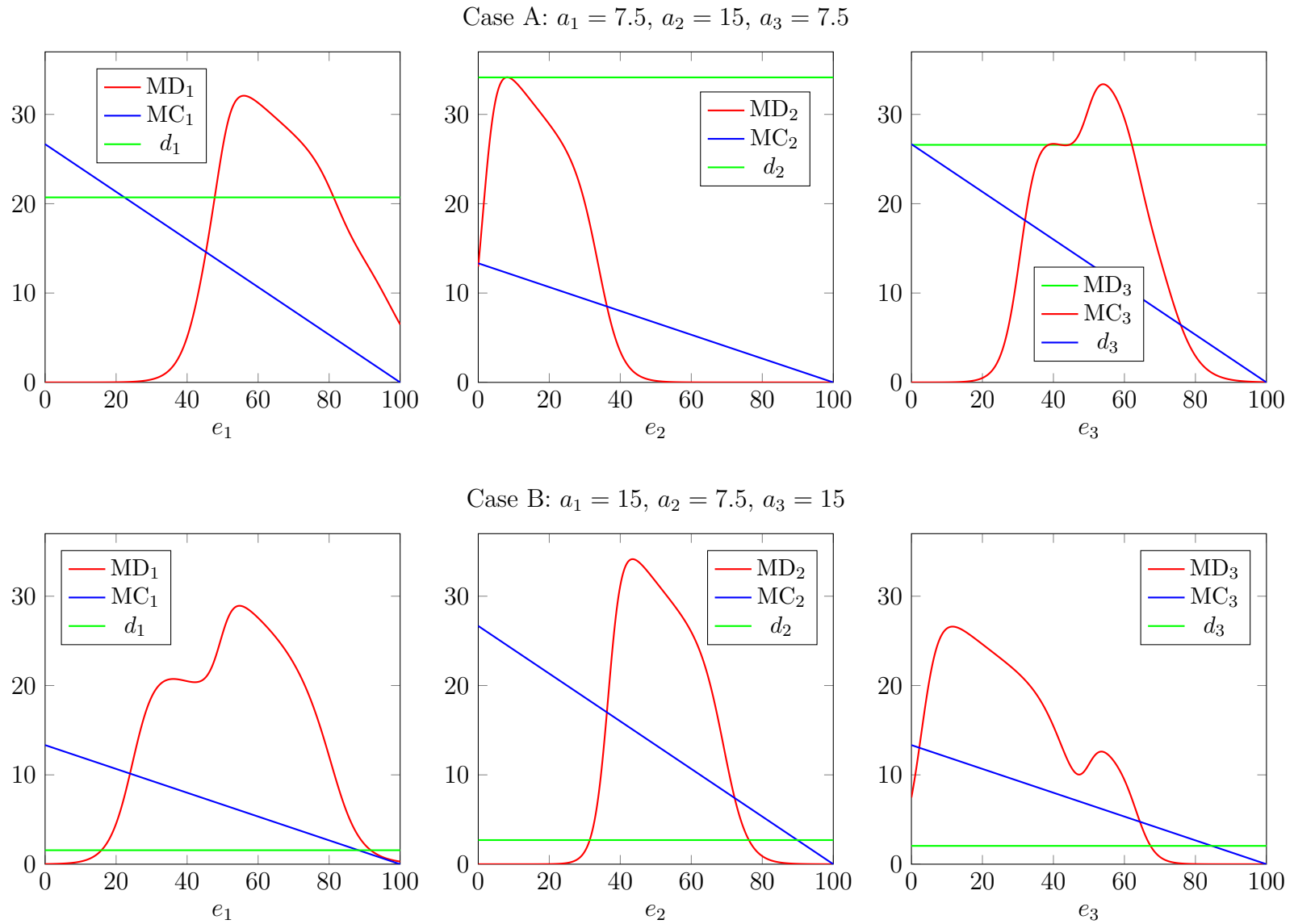


Figure 2: Marginal damages, marginal costs, and trading coefficients. Top panels: Case A ($a_1 = 7.5, a_2 = 15, a_3 = 7.5$). Bottom panels: Case B ($a_1 = 15, a_2 = 7.5, a_3 = 15$). Horizontal axis measures emissions for each source, with other sources' emissions held at the efficient levels.

589 at the social optimum. Lastly, note that Figure 2 illustrates that with nonlinear damages the
590 first-order conditions are not sufficient. In the upper right panel we see that the marginal damage
591 curve for source 3 crosses its marginal cost curve twice. Also, though not plotted, each source
592 has infinitely many marginal damage curves corresponding to different emissions levels by other
593 sources.

594 **Case B:** In this case, a high-cost firm is located downstream of two low-cost firms. At the social
595 optimum, source 3 abates all of its emissions while source 2 emits the most (Table 2). Efficient
596 trades should occur under the TRS, because this system allows the high-cost downstream source
597 to buy permits from the two low-cost upstream sources. Indeed, this is exactly what happened in
598 the simulation. Each firm is allocated 18.5 units of discharge permits initially in both the TRS and
599 the DTRS. In the TRS equilibrium, firm 2 bought 14.4 units from source 3 to increase its emissions
600 to 32.9 while firm 3 sold 18.5 units at the trading ratio $\tau_{32} \approx 0.78$ (i.e. $18.5 \times \tau_{32} = 14.4$ units for
601 source 2). Note that under the TRS, the downstream firm has a choice of buying permits from
602 either source 1 or source 3. Therefore, source 2 buys from the partner with the lowest effective
603 permit price.

604 It follows then that the effective equilibrium prices are equalized across space: $p_1 = \tau_{12}p_2 =$
605 $\tau_{32}p_2 = p_3$. At this equilibrium price, source 1 has no incentive to sell its permits to source 2
606 and thus ends up emitting exactly at the initial allocation. The trading outcome in the DTRS
607 is similar. Source 3 abates completely and sells its permits mostly to source 2. A difference
608 occurs, though, because source 3 also sells its permits to source 1. As we have noted, under the
609 DTRS firms located in different branches are allowed to trade, and the equilibrium prices satisfy
610 $p_1 = (d_1/d_2)p_2 = (d_1/d_3)p_3$. It turns out that at these equilibrium prices, it is cheaper for source
611 1 to buy permits from source 3. As a result, source 1's equilibrium emissions are slightly higher
612 under the DTRS than under the TRS while source 2's equilibrium emissions are lower under the
613 DTRS than under the TRS. The extra trade between source 1 and source 3, however, decreased
614 efficiency slightly compared to the TRS equilibrium. This is because the damage-denominated
615 trading coefficients, the d_i 's, are poor approximations of the true marginal damages at the optimum,
616 as shown in Figure 2. In this case, therefore, the DTRS encouraged inefficient trades. Note, however,
617 that both TRS and DTRS reduce deadweight loss relative to the no-trading baseline, and closely
618 approximate the social optimum in this case.

619 **6.3 Prices vs. quantities**

620 On one hand, our numerical analysis suggests the impossibility of getting prices right in watersheds
621 with branches and nonlinearities that significantly influence marginal damages. Neither the TRS

622 nor the DTRS succeeds in providing the correct price signals. On the other hand, our analysis
 623 also indicates that in some cases the equilibria approximate the social optimum quite closely. We
 624 obtained these results by setting the total supply of permits equal to the socially optimal level.
 625 A natural question then is, which causes the greater efficiency loss: not getting the quantity of
 626 permits right, or not getting the price of permits right? We investigate this question by simulating
 627 the trading outcomes for varying levels of the total supply of permits (in percentage reduction from
 628 the baseline discharge level) and again allocating permits in equal number to each discharger.

629 In order to make the relevant comparison we compute two measures of efficiency loss. Let
 630 $\mathbf{a}^T(A)$ denote the equilibrium abatement vector that emerges under trading scheme T , which can
 631 be either the TRS or the DTRS. In either case, the equilibrium is subject to the constraint that
 632 $\sum \bar{L}_i = A$ for some value A . The first measure is efficiency loss when the quantity of permits is
 633 correct, that is, equal to the sum of efficient emissions levels. It is given by

$$L_T^{\text{eff}} = \frac{[C(\mathbf{a}^T(\sum e_i^{\text{eff}})) + D(\mathbf{a}^T(\sum e_i^{\text{eff}}))] - [C(\mathbf{a}^{\text{eff}}) + D(\mathbf{a}^{\text{eff}})]}{[C(\mathbf{a}^{\text{eff}}) + D(\mathbf{a}^{\text{eff}})]}. \quad (16)$$

634 The first term in the numerator is social cost after trading when the total number of permits is equal
 635 to the efficient level of total emissions. The denominator is social cost at the efficient outcome.

636 The second measure is the maximum efficiency loss due to mis-specifying the total supply of
 637 permits, which is given by

$$L_T^{\text{max}} = \max_{0 \leq A \leq \sum_i \bar{e}_i} \frac{[C(\mathbf{a}^T(A)) + D(\mathbf{a}^T(A))] - [C(\mathbf{a}^{\text{eff}}) + D(\mathbf{a}^{\text{eff}})]}{[C(\mathbf{a}^{\text{eff}}) + D(\mathbf{a}^{\text{eff}})]}. \quad (17)$$

638 The results are shown in Figure 3, where the dashed light blue line in the rightmost panels
 639 is drawn at the minimum of $C + D$. The monetary value corresponds to the the efficient level of
 640 aggregate emissions, in both cases about 80% of the initial total. First, the relative performance
 641 of the two systems varies, in an unsystematic way, with the supply of permits. In Case A, where a
 642 low-cost source is located downstream of two high-cost sources, the DTRS performed substantially
 643 better than TRS when the total supply of permits was kept to the socially optimal level. This is
 644 because the TRS prohibited any trade from taking place. (The curves with no trading coincide
 645 with the TRS curves in Case A.) However, when the total supply of permits is reduced to 60-70%
 646 of the baseline discharge level, the TRS performs better than the DTRS despite the fact that no
 647 trading still takes place under the TRS. This occurs because the efficiency loss due to the TRS
 648 precluding trading was outweighed by the efficiency loss due to the DTRS encouraging inefficient
 649 trades, which increased environmental damages substantially relative to no trade.

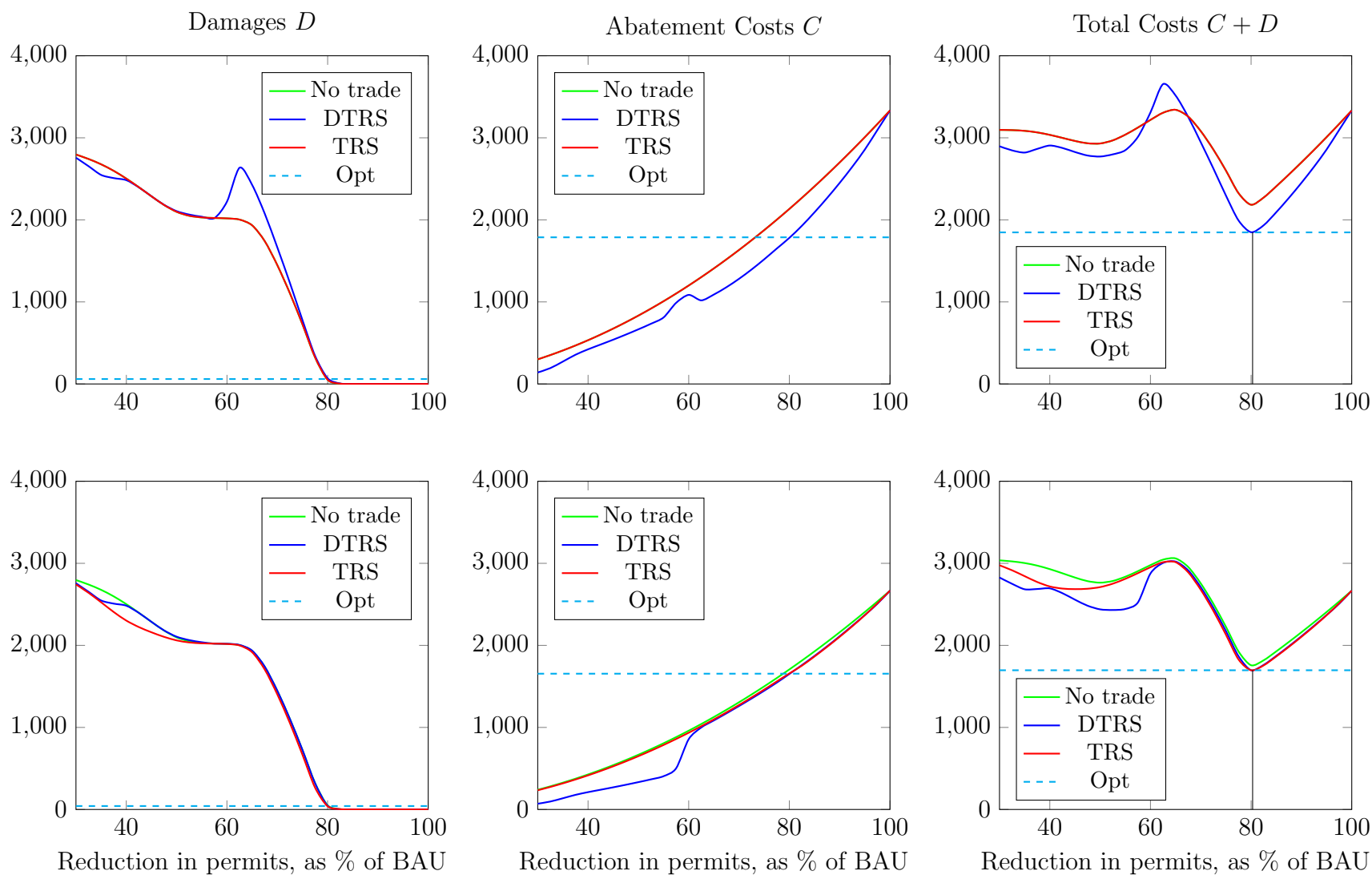


Figure 3: Performance of TRS and DTRS compared. Top panels: Case A ($a_1 = 7.5, a_2 = 15, a_3 = 7.5$). Bottom panels: Case B ($a_1 = 15, a_2 = 7.5, a_3 = 15$). Horizontal axis measures total permit allocation as a percent of business-as-usual emissions total.

650 In contrast, in Case B, in which a high-cost source is located downstream of two low-cost
651 sources, the DTRS performed slightly better than the TRS for all levels of initial permit supplies.
652 In this case, TRS and DTRS provide similar price signals so that the magnitude of the efficiency
653 loss due to environmental damages is similar between the two systems (see the left panel of Case B
654 in Figure 3). However, because the DTRS offers more flexibility in trading, it reduces abatement
655 costs a bit more than does the TRS. This effect dominates the relative performance of the two
656 systems.

657 Second, total economic costs $C + D$ do not exhibit a simple convex relationship with respect
658 to the total supply of permits under the two systems. This is because neither environmental
659 damages nor abatement cost has a simple relationship to the supply of permits. Despite the
660 fact that environmental damages are defined as a decreasing function of emissions or pollution
661 concentrations, environmental damages in the trading equilibrium are not necessarily a decreasing
662 function of the *reduction in the total supply of permits*, and analogously, despite the fact that
663 abatement costs are a convex function of *abatement levels*, total abatement costs are not necessarily
664 a convex function of the *reduction in the total supply of permits*. These effects are especially
665 strong in the DTRS, because the damage-denominated trading coefficients are based on marginal
666 damages at the initial allocation, and thus depend endogenously on the initial supply of permits.
667 Somewhat counter-intuitively, these trading coefficients can either decrease or increase efficiency
668 relative to the TRS. On one hand, the trading coefficient can decrease efficiency by providing
669 incorrect trading margins, which adversely affects environmental damages. On the other hand,
670 however, the trading coefficients can improve efficiency by providing flexibility for trading partners,
671 which reduces abatement costs.

672 Lastly, at least in the current model, getting the quantity of permits right appears to be more
673 important than getting the prices of permits right. In Case A, the estimated efficiency losses,
674 computed according to equation (16), are only 18.2% and 0.1% of the total economic damages,
675 respectively, for TRS and DTRS when the socially optimal number of permits is issued. (In Case B,
676 the corresponding results are 3.6% for TRS and 0.03% for DTRS.) In contrast, the maximum
677 efficiency losses due to mis-specifying the total supply of permits, computed according to equation
678 (17), are 80.9% and 97.8% of the total economic damages, respectively, for TRS and DTRS. (In
679 Case B, the corresponding results are 80.3% for TRS and 78.1% for DTRS.) It is important to
680 emphasize that this result is not a direct consequence of the logistic damage response we assumed
681 in (14). Rather it stems from the simultaneous effects of mis-specifying the quantity of pollution
682 as well as the incorrect price signals that arise from it.

7 Discussion

684 In this paper we examined the efficiency properties of two recently developed water-quality trading
685 models, the trading ratio system (TRS) proposed in Hung and Shaw (2005) and the damage-
686 denominated trading ratio system (DTRS) proposed in Farrow *et al.* (2005). We showed that
687 neither system is sure to achieve the cost-effectiveness optimum (and thus the efficient outcome).
688 More specifically, the TRS encounters difficulties when the river has critical zones at a confluence.
689 The DTRS encounters difficulties when damages are nonlinear. We derived these results under the
690 first-best scenario in which the regulator knows the efficient vector of environmental constraints
691 (for TRS) and the efficient damage constraint (for DTRS). Furthermore, in a second-best scenario
692 where the regulator cannot set these constraints at the efficient levels, neither system dominates in
693 terms of efficiency, because the TRS excludes efficient trades while the DTRS promotes inefficient
694 trades. In this sense our results indicate the impossibility of getting the spatially explicit prices of
695 pollution right under either system.

696 Our computational model also contains some encouraging findings: the welfare losses associated
697 with the two systems are dwarfed by those associated with choosing the incorrect number of permits.
698 Thus, getting quantities right is more important than getting prices right. The two effects interact
699 in interesting ways, though. The magnitude of inefficiency due to incorrect price signals appears
700 to depend on the total supply of permits. Thus our paper suggests the importance of getting the
701 quantity of pollution right even while striving to get the spatial prices of pollution right.

702 Our main message, then, is that either scheme can perform well so long as the aggregate supply
703 of permits is set carefully. We have not provided an alternative scheme that addresses the difficulties
704 we have found. That project is left for future work and we believe fruitful work might lie along
705 either of two lines. One is to investigate empirically the geographic distribution of polluting sources
706 and the degree and nature of nonlinearity in environmental damages in actual watersheds, and for
707 key water pollutants. It may be the case that the concerns raised here are minor in comparison to
708 the benefits conferred by improved water quality.

709 Second, Antweiler (2012) has developed a powerful iterative approach to permit-trading policy
710 for air quality. The idea of the scheme is for the regulator to adjust the number of permits issued
711 each period, based on each polluter's marginal contribution to environmental damages at its current
712 emissions level. As long as social welfare is globally concave, this iterative process would eventually
713 converge to the social optimum. Our conjecture is that this iterative scheme could potentially be
714 embedded in either the TRS or the DTRS. Future studies might investigate the performance of the
715 iterative approach, with any of the three schemes, for watersheds characterized by both nonlinear

716 damages and branching rivers.

717 **Appendix**

718 *Proof of Proposition 1.* Suppose that \mathbf{e}^{HS} is the solution vector for program (1). By assumption,
 719 we must have a critical zone at the confluence receptor m : $\sum_{m-1_i} \tau_{(m-1_i)m} \bar{X}_{m-1_i} > \bar{X}_m$, where
 720 $\{m-1_i\}_i$ is the collection of indices immediately upstream of zone m , along two or more branches.
 721 We know that at the optimum, given our assumption that $C'_i > 0$,

$$\bar{X}_m = \sum_{m-1_i} \tau_{(m-1_i)m} e_{m-1_i}^{\text{HS}}.$$

722 Suppose, without loss of generality, that there are only two zones upstream of the critical confluence,
 723 one on each of two tributaries. Call these zones a and b . By assumption,

$$\bar{X}_m = \tau_{am} e_a^{\text{HS}} + \tau_{bm} e_b^{\text{HS}}.$$

724 On the other hand, without knowing individual sources' cost functions, information on \mathbf{e}^{HS} is not
 725 available to the TRS regulator. Therefore she must, without knowledge of \mathbf{e}^{HS} , allocate zonal
 726 discharge load standards \bar{Z} 's such that $\bar{Z}_m = 0$ and

$$\bar{X}_m = \tau_{am} \bar{Z}_a + \tau_{bm} \bar{Z}_b.$$

727 Because there is only one constraint equation for two zonal standards, the allocation is indetermi-
 728 nate. It is trivial to see that if the TRS allocates \bar{Z} 's in such a way that, for example, $e_a^{\text{HS}} > \bar{Z}_a$
 729 and $\bar{Z}_b = (\bar{X}_m - \tau_{am} \bar{Z}_a) / \tau_{bm} > e_b^{\text{HS}}$, then the trading equilibrium can never achieve \mathbf{e}^{HS} . Similar
 730 arguments apply when there are more than two upstream zones. This completes the proof. \square

731 *Proof of Proposition 2.* We offer a proof for the case of an interior optimum of (2). Note that
 732 at the interior optimum, the emission vector \mathbf{e}^{FSCH} must satisfy the necessary (but not sufficient)
 733 condition:

$$\frac{\partial D(\mathbf{e}^{\text{FSCH}}) / \partial e_i}{\partial D(\mathbf{e}^{\text{FSCH}}) / \partial e_j} = \frac{C'_i(a_i^{\text{FSCH}})}{C'_j(a_j^{\text{FSCH}})} \quad \text{for all } i, j,$$

734 where $e_i^{\text{FSCH}} = \bar{e}_i - a_i^{\text{FSCH}}$. On the other hand, according to equation (9), at an interior equilib-
 735 rium, we have

$$\frac{d_i}{d_j} = \frac{C'_i(a_i^{\text{FSCH}})}{C'_j(a_j^{\text{FSCH}})} \quad \text{for all } i, j.$$

736 Thus, in order for the trading equilibrium to achieve the cost-effective solution, the regulator must
 737 evaluate the exchange rates (the d 's) at the optimum: $d_i^{\text{FSCH}} = \partial D(\mathbf{e}^{\text{FSCH}}) / \partial e_i$. Under the

738 DTRS, the regulator allocates \bar{L} 's in such a way that:

$$\sum_i d_i^{\text{FSCH}} \bar{L}_i = \overline{\text{TD}} = D(\mathbf{e}^{\text{FSCH}}). \quad (18)$$

739 We now ask whether there exists some initial allocation $\bar{\mathbf{L}}$, satisfying (18), such that the resulting
 740 equilibrium would achieve the cost-effective solution. We claim that such an allocation does not
 741 exist. Suppose, by way of contradiction, there exists such an allocation $\bar{\mathbf{L}}$ and that the resulting
 742 trading equilibrium is also cost-effective: $\mathbf{e}^{\text{DTRS}} = \mathbf{e}^{\text{FSCH}}$. Because the equilibrium must satisfy
 743 the market-clearing condition (10b), we have

$$\sum_i d_i^{\text{FSCH}} \bar{L}_i = \sum_i d_i^{\text{FSCH}} e_i^{\text{DTRS}}. \quad (19)$$

744 However, because the aggregate damage function is nonlinear, we have

$$\sum_i d_i^{\text{FSCH}} e_i^{\text{FSCH}} \neq D(\mathbf{e}^{\text{FSCH}}). \quad (20)$$

745 Combining (18), (19), and (20), we see that

$$\sum_i d_i^{\text{FSCH}} \bar{L}_i = \sum_i d_i^{\text{FSCH}} e_i^{\text{DTRS}} = D(\mathbf{e}^{\text{FSCH}}) \neq \sum_i d_i^{\text{FSCH}} e_i^{\text{FSCH}}.$$

746 which contradicts that $\mathbf{e}^{\text{DTRS}} = \mathbf{e}^{\text{FSCH}}$. This completes the proof. \square

747 *Proof of Proposition 3.* To see (i), given the efficient solution \mathbf{a}^{eff} , let the constraint vector $\bar{\mathbf{X}}^{\text{eff}}$
 748 be defined as

$$\bar{\mathbf{X}}^{\text{eff}} = T(\bar{\mathbf{e}} - \mathbf{a}^{\text{eff}})'$$

749 Suppose by way of contradiction that \mathbf{a}^{HS} solves (1) subject to $\bar{\mathbf{X}} = \bar{\mathbf{X}}^{\text{eff}}$, but $\mathbf{a}^{\text{HS}} \neq \mathbf{a}^{\text{eff}}$. Because
 750 D is increasing in \mathbf{x} , and because $\mathbf{x}^{\text{eff}} = T(\bar{\mathbf{e}} - \mathbf{a}^{\text{eff}})' = \bar{\mathbf{X}}^{\text{eff}}$, we have

$$\sum_i C_i(a_i^{\text{eff}}) + D(\mathbf{x}^{\text{eff}}) < \sum_i C_i(a_i^{\text{HS}}) + D(\mathbf{x}^{\text{HS}}) \leq \sum_i C_i(a_i^{\text{HS}}) + D(\mathbf{x}^{\text{eff}}),$$

751 where the first inequality follows from $\mathbf{a}^{\text{HS}} \neq \mathbf{a}^{\text{eff}}$ and the second inequality follows because $\mathbf{x}^{\text{HS}} \leq$
 752 \mathbf{x}^{eff} implies $D(\mathbf{x}^{\text{HS}}) \leq D(\mathbf{x}^{\text{eff}})$. But this inequality implies that there exists $\mathbf{a}^{\text{eff}} \neq \mathbf{a}^{\text{HS}}$ such that
 753 $\sum_i C_i(a_i^{\text{eff}}) < \sum_i C_i(a_i^{\text{HS}})$ with $\mathbf{x}^{\text{eff}} = \bar{\mathbf{X}}^{\text{eff}}$, a contradiction.

754 The proof of (ii) is analogous, with the constraint value $\overline{\text{TD}}^{\text{eff}}$ defined as $\overline{\text{TD}}^{\text{eff}} = D(\bar{\mathbf{e}} - \mathbf{a}^{\text{eff}})$.

755 \square

756 *Proof of Proposition 6.* (i) Let $\bar{\mathbf{Z}}^{SB}$ denote the second-best allocation of permits under the TRS

757 and define $\bar{\mathbf{X}}^{SB} = T\bar{\mathbf{Z}}^{SB}$. In a branchless river, Hung and Shaw's result applies and so the TRS
 758 equilibrium achieves the solution \mathbf{a}^{SB} to the cost-effective program (1) given \mathbf{X}^{SB} . By definition,
 759 \mathbf{a}^{SB} is not the same as \mathbf{a}^{eff} . Thus there exists some abatement vector \mathbf{a}' such that

$$\sum_i C_i(a_i^{SB}) + D(\mathbf{x}^{SB}) > \sum_i C_i(a'_i) + D(\mathbf{x}') \geq \sum_i C_i(a_i^{\text{eff}}) + D(\mathbf{x}^{\text{eff}}).$$

760 On the other hand, an interior DTRS equilibrium satisfies

$$\frac{C'_i(a_i^{DTRS})}{C'_j(a_j^{DTRS})} = \frac{d_i}{d_j} = \frac{p_i}{p_j}, \quad (21a)$$

$$\sum d_i \bar{L}_i^{SB} = \sum d_i (\bar{e}_i - a_i^{DTRS}). \quad (21b)$$

761 Here the notation a_i^{DTRS} is distinct from our earlier a_i^{FSCH} in that the former, employed in equations
 762 (15), may not solve (2). Now, replace \mathbf{a}^{DTRS} with \mathbf{a}' . It is easy to see that given $(\bar{\mathbf{L}}^{SB}, \bar{\mathbf{e}}, \mathbf{a}')$, this
 763 system of equations can be solved for \mathbf{d} . It follows then that there exists a vector of trading ratios
 764 \mathbf{d} such that the DTRS equilibrium arising from $(\bar{\mathbf{L}}^{SB}, \bar{\mathbf{e}}, \mathbf{d}')$ achieves \mathbf{a}' .

765 (ii) Define $\overline{\text{TD}}^{SB} = S(\bar{\mathbf{X}}^{SB}) = D(\bar{\mathbf{L}}^{SB})$. We know from Farrow *et al.* that when damages
 766 are linear, the DTRS mechanism achieves the solution \mathbf{a}^{SB} to the cost-effective program (2) given
 767 $\overline{\text{TD}}^{SB}$. On the other hand, there exists a cost-effective program (1) that achieves \mathbf{a}^{SB} . Proposition 1
 768 shows that the TRS scheme may fail to achieve \mathbf{a}^{SB} if the branching river has a critical zone at the
 769 confluence. Now, unlike in part (i), the TRS requires polluting sources to still obey $\bar{\mathbf{X}}^{SB}$. It thus
 770 follows that

$$\sum_i C_i(a_i^{TRS}) + D(\mathbf{x}^{TRS}) > \sum_i C_i(a_i^{DTRS}) + D(\mathbf{x}^{DTRS}) = \sum_i C_i(a_i^{SB}) + D(\mathbf{x}^{SB}).$$

771 This completes the proof. □

772 References

- 773 [1] Anderson, D.M, P.M. Glibert, J.M. Burkholder. (2002). Harmful Algal Blooms and Eutrophication: Nutrient Sources, Composition, and Consequences. *Estuaries*, 25(4), 704–726
774
- 775 [2] Antweiler, Werner, “Emission Permit Trading for Air Pollution Hot Spots,” *Working paper*,
776 University of British Columbia, (September, 2012).
- 777 [3] Baumol, W.J., W.E. Oates. (1988). *The Theory of Environmental Policy*. 2nd Edition. Cambridge: Cambridge University Press.
778
- 779 [4] Carlson, Curtis, Dallas Burtraw, Maureen Cropper, and Karen L. Palmer. (2004). Sulfur dioxide control by electric utilities: What are the gains from trade?” *Journal of Political Economy*, 108, 1292–1326.
780
781
- 782 [5] Elliott, J.M. (2000). Pools as refugia for brown trout during two summer droughts: trout responses to thermal and oxygen stress. *Journal of Fish Biology*, 56, 938–948.
783
- 784 [6] Farrow, R.S, Shultz, M.T., Celikkol, P. Van Houtven, G.L. (2005). Pollution Trading in Water Quality Limited Areas: Use of Benefits Assessment and Cost-Effective Trading Ratios. *Land Economics*, 81 (2), 191–205.
785
786
- 787 [7] Fisher-Vanden, Karen and Sheila Olmstead, “Moving Pollution Trading from Air to Water: Potential, Problems, and Prognosis,” *Journal of Economic Perspectives*, 27 (2013), 147–172.
788
- 789 [8] Hung, Ming-Feng and Dagee Shaw. (2005). A trading-ratio system for trading water pollution discharge permits. *J. of Env. Economics & Mgmt.*, 49, 83–102.
790
- 791 [9] King, Denis M. and Peter J. Kuch. (2003). Will nutrient credit trading ever work? An assessment of supply and demand problems and institutional obstacles. *The Environmental Law Reporter*. Washington, DC: Environmental Law Institute.
792
793
- 794 [10] Krupnick, A.J., W.E. Oates, and E. Van De Verg (1983), On marketable air-pollution permits: The case for a system of pollution offsets, *J. Environ. Econ. Mgmt.*, 10, 233–247.
795
- 796 [11] McGartland, A.M. and W.E. Oates (1985), Marketable permits for the prevention of environmental deterioration, *J. Environ. Econ. Mgmt.*, 12, 207–228.
797
- 798 [12] Montgomery, W. David. (1972). Markets in licenses and efficient pollution control programs. *Jour. of Economic Theory*, 5, 395–418.
799

- 800 [13] Morgan, Cynthia and Ann Wolverton. (2005). Water quality trading in the United States.
801 Working paper #05-07, U.S. EPA National Center for Environmental Economics, Wathington
802 DC: EPA.
- 803 [14] Muller, N.Z., Mendelsohn, R. (2009). Efficient Pollution Regulation: Getting the Prices Right.
804 *American Economic Review*, 99(5), 1714–1739.
- 805 [15] Schnoor, J.L. (1996). *Environmental modeling: Fate and transport of pollutants in water, air,*
806 *and soil*. Wiley: New York.
- 807 [16] Todd, D.K. and L.W. Mays. (2005). *Groundwater Hydrology*. John Wiley & Sons, Inc. 3rd
808 edition.
- 809 [17] U.S. EPA. (2002). Estimation of national economic benefits using the National Water Pollution
810 Control Assessment Model to evaluate regulatory options for concentrated animal feeding
811 operations. U.S. EPA Office of Water, Washington DC: EPA. EPA-821-R-03-009.
- 812 [18] U.S. EPA. (2003). Water quality trading policy. U.S. EPA Office of Water, Washington DC:
813 EPA.
- 814 [19] U.S. EPA. (2004). Water quality trading assessment handbook: Can water quality trading
815 advance your watershed’s goal? U.S. EPA Office of Water, Washington DC: EPA.
- 816 [20] Vermillion River Watershed Joint Powers Board. (2008). Findings and recommenda-
817 tions for stabilizing stream temperature and volume (last accessed Mar. 19, 2013).
818 [www.vermillionriverwatershed.org/index.php?option=com_content&view=article&id=52&](http://www.vermillionriverwatershed.org/index.php?option=com_content&view=article&id=52&itemid=61)
819 [itemid=61](http://www.vermillionriverwatershed.org/index.php?option=com_content&view=article&id=52&itemid=61).