RIMS Workshop 2024
Analytic Number Theory and Related Topics

Organizers: Maki Nakasuji (Sophia University / Tohoku University)
Takashi Taniguchi (Kobe University)

Date: October 15 (Tue) 9:50 – October 18 (Fri) 16:30, 2024
Place: Room 420, RIMS, Kyoto University, Japan

Abstracts

October 15 (Tue)

10:00 – 10:40 **Hajime Kaneko (Tsukuba University)**
"Descriptive complexity results related to uniform distribution theory"
(joint work with Bill Mance)

The Borel hierarchies measure the complexity of Borel sets. As an example related to analytic number theory, Ki and Linton determined the Borel hierarchy of the set of normal numbers in base b, where b is a fixed integer greater than 1. The normality of real numbers in base b is related to the uniform distribution properties of the fractional parts of geometric sequences with common ratio b.

Although the uniformity of fractional parts of geometric sequences is an important problem of uniform distribution theory, their properties of Borel hierarchies are generally unknown if the common ratios are not integers. In this talk, we introduce results on the Borel hierarchies when common ratios are Pisot numbers. Moreover, we discuss the fractional parts of general linear recurrence sequences.

This is a joint work with Bill Mance.

11:00 – 11:30 **Yuichiro Toma (Nagoya University)**
"Recovering short generators via negative moments of Dirichlet L-functions"
(joint work with Iu-Iong Ng)

Cryptographic schemes based on lattice problems are considered to be one of the candidates for quantum computation-resistant cryptosystems. Some algebraic ideal lattice-based cryptosystems rely on the hardness of computing short generators of a given principal ideal. In EUROCRYPT 2016, Cramer, Ducas, Peikert, and Regev proposed an efficient algorithm for recovering short generators of principal ideals in q-th cyclotomic fields with q being a prime power.

In this talk, we present the results of improving the algorithm given by Cramer, Ducas, Peikert, and Regev for the case when q is prime by calculating the negative square moments of the Dirichlet L-functions at $s = 1$. This is a joint work with Iu-Iong Ng (Nagoya University).

11:50 – 12:20 **Yuto Tsuruta (Tohoku University)**
"A q-analogue of Maesaka–Seki–Watanabe's formula and its application"

It is well known that the multiple zeta value $\zeta(k_1, \dots, k_r)$ has an iterated integral expression:

$$\int_{0<t_{11}<\cdots<t_{1k_1}<\cdots<t_{r1}<\cdots<t_{rk_r}<1} \frac{\mathrm{d}t_{11}}{1-t_{11}}\frac{\mathrm{d}t_{12}}{t_{12}}\cdots\frac{\mathrm{d}t_{1k_1}}{t_{1k_1}}\cdots\frac{\mathrm{d}t_{r1}}{1-t_{r1}}\frac{\mathrm{d}t_{r2}}{t_{r2}}\cdots\frac{\mathrm{d}t_{rk_r}}{t_{rk_r}}.$$

This workshop is partially supported by RIMS and JSPS KAKENHI Grant Number 22K03274, 23K22386.

Maesaka, Seki, and Watanabe recently found a fascinating phenomenon called the MSW formula. It gives a discretization of iterated integral expression of the multiple zeta value. Furthermore, Yamamoto extended MSW formula to Schur multiple zeta values of diagonally constant indices. In this talk, we give a q-analogue of MSW formula by considering Yamamoto's work and alternative proof of the relation among finite multiple harmonic q-series at roots of unity.

13:50 – 14:20 **Yuta Kadono (Tohoku University)**

"On the Hurwitz-Lerch type central binomial series and the bivariate Eulerian polynomial "
(joint work with Karin Ikeda)

The central binomial series (CBS) is a type of Dirichlet series that features a central binomial coefficient in the summand. Its special values at integer points have also been studied in connection with various objects. In particular, Lehmer clarified the link between the polylogarithmic version of CBS and the arcsine function, and investigated the special values of CBS at integer points. This was later given an interpretation with bivariate Eulerian polynomials by Bényi and Matsusaka. In this talk, we introduce 'Hurwitz type CBS (HCBS)', which adds a real parameter to CBS. Then, a generalization of the above results is given by considering a polylogarithmic version of HCBS (called Hurwitz-Lerch type CBS) and its representation using generalized hypergeometric functions. This talk is based on ongoing joint work with Karin Ikeda of Kyushu University.

14:40 – 15:20 **Siddhi Pathak (Chennai Mathematical Institute)**

"Zeros of polynomials related to special values of L-functions"
(joint work with Mrityunjoy Charan)

L-functions are central objects in number theory and their special values are expected to capture significant arithmetic data. An interesting theme is to study polynomials whose coefficients arise from special values, for instance, period polynomials associated with Hecke newforms. It has been recently shown that all the zeros of these period polynomials lie on the unit circle. Same is also true for all the non-real zeros of Ramanujan polynomials i.e, those associated to products of values of the Riemann zeta-function at positive even integers. In this talk, we discuss this theme and establish that all non-real zeros of certain variants of Ramanujan polynomials lie on the unit circle, proving a conjecture of B. Maji and T. Sarkar.

15:40 – 16:40 **Robert Lemke Oliver (Tufts University)**

Enumerating Galois extensions of number fields

Let k be a number field. We provide an asymptotic formula for the number of Galois extensions of k with absolute discriminant bounded by some $X \geq 1$ as $X \to \infty$. We also provide an asymptotic formula for the closely related count of extensions K/k whose normal closure has discriminant bounded by X. The key behind these results is a new upper bound on the number of Galois extensions of k with a given Galois group G and discriminant bounded by X; we show the number of such extensions is $O_{[k:Q],G}(X^{4/\sqrt{|G|}})$. This improves over the previous best bound $O_{k,G,\epsilon}(X^{3/8+\epsilon})$ due to Ellenberg and Venkatesh. In particular, ours is the first bound for general G with an exponent that decays as $|G| \to \infty$.

October 16 (Wed)

9:30 – 10:10 **Hiro-aki Narita (Waseda University)**

"Sup norm bounds for some theta lifts to $O(1, 8n + 1)$"
(joint work with Simon Marshall and Ameya Pitale)

It is a fundamental problem to understand distributions of Laplace eigenvalues for Riemannian manifolds, whose studies are motivated by geometry, analysis (geometric analysis) and quantum physics etc. One related important problem is to give sup norm bounds for Laplace eigenfunctions in terms of powers of their Laplace eigenvalues. The aim of this talk is to take up this problem for cusp forms explicitly given by theta lifts from Maass cusp forms of level one to cusp forms on $O(1, 8n + 1)$. In the setting of automorphic forms, sup norm bounds were studied mainly for compact locally symmetric spaces or restriction to compact subsets. Our sup norm bounds hold "globally" on a locally symmetric space of non-compact type (for $O(1, 8n + 1)$ or real hyperbolic space of dim$= 8n + 1$), namely not restriction to compact subsets. A point of our upper bound is that its exponent is not equal to but closed to so called "Sarnak bound" and that the difference is understood in terms of some quantity related to the Kim-Sarnak bound for the Ramanujan conjecture of Maass cusp forms.

10:30 – 11:10 **Andrei Seymour-Howell (University of Bristol)**

"Murmurations of Maass forms"
(joint work with Andrew R. Booker, Min Lee, David Lowry-Duda and Nina Zubrilina)

In 2022 He, Lee, Oliver and Pozdnyakov made an interesting discovery, using techniques from machine learning, of a correlation between the root number of a family of elliptic curves and the coefficients of the associated L-functions. They named this phenomenon "murmurations of elliptic curves" due to its visual similarities to the phenomenon in birds. Subsequently, murmurations in many other families of objects have been discovered, namely Dirichlet characters and holomorphic modular forms. All these objects have an arithmetic analogue which begs the question, is this an arithmetic phenomenon? In this talk I will demonstrate that this phenomenon is also true for Maass cusp forms, showing that this phenomenon is also occurring for non-arithmetic objects. This is joint work with Andrew R. Booker, Min Lee, David Lowry-Duda and Nina Zubrilina.

11:30 – 12:00 **Satoshi Tsuchimi (Kobe University)**

"A multivariate analogue of the Zwegers' μ-function"
(joint work with Genki Shibukawa)

The μ-function introduced by Zwegers in 2002 is a generalization of the mock theta functions discovered by Ramanujan in 1992. In this talk, we introduce a multivariable analogue of the μ-function from a divergent solution appearing in a factorized q-difference equation. Then we give some properties satisfied by this function. This is a joint work with Genki Shibukawa (Kobe University).

13:40 – 14:20 **Yoshinori Mizuno (Nagoya Institute of Technology)**

"Mitsui's Dirichlet series for Hermitian matrices'
(joint work with Roland Matthes)

To develop the theory of partition in matrix setting, Takayoshi Mitsui recognized the importance of the Dirichlet series constructed from traces of positive-definite matrices, and raised the problem to establish an analytic continuation of the series (1982). Egami and Sato (1998) announced the solution of this problem for the lattice consisting of 2×2 positive-definite integral symmetric matrices, which is the index set of summands defining the Dirichlet series.

But details are still unpublished. The same series (with half-integral symmetric matrices) was studied in the dissertation of Diehl (2011, Imamoḡlu's student), but it was not a complete solution because certain conditions about the Shintani zeta function are imposed. Independent of Mitsui's problem, Duke and Imamoḡlu (2004) investigated a similar Dirichlet series with the motive of counting integral points in cones and gave an analytic continuation of the series. Duke and Imamoḡlu (and also Egami (1994)) treated the lattice whose automorphism group is a cocompact subgroup of $\mathrm{SL}_2(\mathbb{R})$ in contrast to the case of $\mathrm{SL}_2(\mathbb{Z})$ treated by Egami-Sato, Diehl.

In this lecture, we solve the Mitsui problem of 2×2 positive-definite half-integral Hermitian matrix in analogy to the case of symmetric matrices. It is achieved with the help of spectral theory of automorphic functions on 3-dimensional upper half-space $\mathbb{H}^3$, a Katok-Sarnak type correspondence, theory of zeta functions of indefinite quadratic forms, an explicit formula of a certain prehomogeneous zeta function, etc. Although it is limited to the case of the Gaussian number field $\mathbb{Q}(i)$, the automorphism group of the lattice defining the Dirichlet series is $\mathrm{SL}_2(\mathbb{Z}[i])$, which is not a cocompact subgroup of $\mathrm{SL}_2(\mathbb{C})$, and thus it is necessary to consider the contribution from the continuous part of $L^2(\mathrm{SL}_2(\mathbb{Z}[i]) \setminus \mathbb{H}^3)$.

14:40 – 15:20 **Kenta Endo (National Institute of Technology, Suzuka College)**

"Limit theorem for the hybrid universality theorem of zeta-functions"

In 1979, Gonek showed the hybrid joint universality theorem for the Dirichlet L-functions and proved the universality theorem for the Hurwitz zeta-functions with rational parameters as its application. The notion of the hybrid universality combines the Voronin's universality theorem with the Kronecker-Wyle approximation theorem. Since the introduction of the hybrid universality theorem, several generalizations, refinements, and applications have been developed. Despite these developments, no probabilistic proof based on Bagchi's approach has been formulated. Bagchi's method is quite a standard method to deduce the universality theorem. In this talk, we present the Limit theorem for the hybrid universality theorem of zeta-functions.

15:40 – 16:40 **Hirofumi Tsumura (Tokyo Metropolitan University)**

'Various types of multiple polylogarithms and related zeta values"
(joint work with Masanobu Kaneko)

In this talk, I will discuss various types of multiple polylogarithms and related multiple zeta values. A logarithmic function is defined as the inverse of an exponential function. From this perspective, we define certain multiple polylogarithms corresponding to generalized exponential functions. As concrete applications, we introduce a class of relation formulas for ordinary multiple polylogarithms, including the five-term relation for the dilogarithm. Additionally, we consider a certain polylogarithm defined as the inverse function of the iterated exponent given by Bell. This report is partly a joint work with Masanobu Kaneko.

October 17 (Thu)

9:30 – 10:10 **Yuta Suzuki (Rikkyo University)**

"Telhcirid's theorem on arithmetic progressions"
(joint work with Gautami Bhowmik)

The classical Dirichlet theorem on arithmetic progressions states that there are infinitely many primes in a given arithmetic progression with a trivial necessary condition. In this talk, we prove a reversed version of this theorem, which may be called Telhcirid's theorem on arithmetic progressions, i.e., we prove that there are infinitely many primes whose reverse of radix representation is in a given arithmetic progression except in some degenerate cases.

Unfortunately, our result needs the base of radix representation to be relatively large. This is a joint work with Gautami Bhowmik (University of Lille).

10:30 – 11:10 **Taka-aki Tanaka (Keio University)**
"Algebraic independence of the values of a certain function invariant under the action of the dihedral group"

Using a suitable linear recurrence of positive integers, we construct a certain function of three complex variables which are invariant under the action of the dihedral group on its domain of definition. For such a function, we consider the map defined on the set of orbits of the action of the dihedral group and its restriction to the set of orbits represented by algebraic points. Then we show that the restriction takes algebraically independent values at any distinct orbits represented by algebraic points.

11:30 – 12:00 **Hayato Kanno (Tohoku University)**
"Multiple Eisenstein series of level N and Goncharov's coproduct for the iterated integrals"

Multiple Eisenstein series (MES) is a holomorphic function on the upper half plane, which is a generalization of double Eisenstein series introduced by Gangl, Kaneko and Zagier. As a connection to multiple zeta value, Bachmann and Tasaka showed that the Fourier expansion of MES is obtained from the coproduct of a Hopf algebra generated by formal iterated integrals corresponding to multiple zeta values. They also constructed shuffle regularized MES. For general level, Yuan and Zhao studied double Eisenstein series of level N and obtained analogous results of Gangl, Kaneko and Zagier. In this talk, we introduce MES of level N and expand the results of Bachmann and Tasaka for arbitrary level. This implies there is a mysterious connection between MES of level N and iterated integrals.

13:40 – 14:10 **Deng Yuqi (Kyushu University)**
"Eichler-Selberg relations for singular moduli"
(joint work with Toshiki Matsusaka and Ken Ono)

The Eichler–Helberg trace formula expresses the trace of Hecke operators on spaces of cusp forms as weighted sums of Hurwitz–Kronecker class numbers. We extend this formula to a natural class of relations for traces of singular moduli, where one views class numbers as traces of the constant function $j_0(\tau) = 1$. More generally, we consider the singular moduli for the Hecke system of modular functions

$$j_m(\tau) := mT_m\left(j(\tau) - 744\right).$$

For each $\nu \geq 0$ and $m \geq 1$, we obtain an *Eichler–Selberg relation*. For $\nu = 0$ and $m \in \{1, 2\}$, these relations are Kaneko's celebrated singular moduli formulas for the coefficients of $j(\tau)$. For each $\nu \geq 1$ and $m \geq 1$, we obtain a new Eichler–Selberg trace formula for the Hecke action on the space of weight $2\nu + 2$ cusp forms, where the traces of $j_m(\tau)$ singular moduli replace Hurwitz–Kronecker class numbers. These formulas involve a new term that is assembled from values of symmetrized shifted convolution L-functions.

14:30 – 15:10 **Kota Saito (Tsukuba University / Sophia University)**
"Mills' constant is irrational"

Let $\lfloor x \rfloor$ be the integer part of a real number x. In 1947, Mills constructed a real number ξ such that $\lfloor \xi^{3^k} \rfloor$ is a prime number for all positive integers k, Mills' constant is the smallest real number $\xi > 1$ satisfying this property. Determining whether the constant is rational or irrational has remained an unsolved problem for a long time. However, we successfully prove

that Mills' constant is irrational. The speaker plans to talk about the historical backgrounds and strategy of proofs. If time permits, the speaker will also introduce partial results on the transcendence of Mills' constant.

15:30 – 16:30 **Jaroslav Hančl (University of Ostrava)**
"Some recent results in Diophantine approximation"

The talk deals with the improvements of famous theorems concerning the continued fractions. It includes, Hurwitz's theorem about approximation of two consecutive partials, Borel's theorem with three consecutive partials, the theorem of Legendre, Vahlen's theorem and others. The substitution of Lagrange numbers by special functions will be also presented.

October 18 (Fri)

9:30 – 10:30 **Timothy Trudgian (UNSW Canberra)**
"Weaker can often be better"

Suppose you need an upper bound on some function $f(x)$ when $x \approx 10^{20}$. Theorem B gives $|f(x)| \leq x$. Theorem C gives $|f(x)| \leq 10x^{0.99}$. C is asymptotically stronger than B, but useless for your purposes.

You could try to improve C, or stick with the weaker B. But why not look at a result that ' s even weaker than B? Call this A, and suppose that A gives $|f(x)| \leq (x \log x)/50$. Although this bound is asymptotically the worst, it is the *best* to use in the application. In this spirit I shall discuss a program of work on the Riemann zeta-function, which aims at obtaining weaker (and, for applications, often better) estimates.

10:50 – 11:30 **Iekata Shiokawa (Keio University)**
"Generalized Stern polynomials and related continued fractions"
(joint work with Daniel Duverney)

We define the generalized Stern polynomials of the type-1 and find the irregular continued fractions with the nth term $z^{t^n}/1$ for $|z| < 1$. In this talk we discuss the continued fractions with the n-th term $e(n)z^{t^n}/1$, where $(e(n))$ is any sequence of 1 or -1 of any length l. We estimate the irrationality exponent of these continued fractions at certain rational points z. Furthermore, we prove the transcendence of these continued fractions for any algebraic z with $0 < |z| < 1$.

11:50 – 12:20 **Keita Nakai (Nagoya University)**
"Joint universality for the Riemann zeta-function with general shifts"

In 2023, Laurinčikas proposed the question of whether the universality theorem for the Riemann zeta-function shifted by an exponential function holds. This problem was solved for more general shifts by Andersson et al. in 2024. In this talk, we will generalize their result for the joint universality theorem for the Riemann zeta-function with general shifts using a different approach from their method.

14:00 – 14:30 **Masaya Kitajima (Nagoya University)**
"Generalized Bessel functions corresponding to lattice point problems of generalized circles"

The lattice point problems of the p-circle (e.g., the astroid), which a generalized circle for positive real numbers p, have been solved for approximately p more than 3 by E. Krätzel and the results of G. Kuba. In this talk, I will discuss the speaker's result that certain functions closely related to the problems can be displayed as series by a newly generalized

Bessel functions based on the property p-radial, generalization of spherical symmetry, and highlight the possibility that attempts to solve the problems via this display are suitable especially for the cases $0 < p \leq 1$. This study is based on the harmonic-analytic method by S. Kuratsubo and E. Nakai, using certain functions generalizing the error term of the circle problem by variables and series representation of the functions by the Bessel functions.

14:50 – 15:30 **Makoto Kawashima (Meiji Gakuin University)**

"Non-vanishing criterion for the p-adic Hurwitz zeta values"
(joint work with Anthony Poëls)

This is a joint work with Anthony Poëls (Lyon University). Little is known about the irrationality of the values of the Kubota-Leopoldt p-adic L-function at positive integer points. In this talk, I will introduce an approach to this problem using the linear independence of special values of the p-adic Hurwitz zeta function. This method was investigated by F. Beukers, P. Bel, J. Sprang and Sprang-L. Lai. Our main result is demonstrating the linear independence of different values of the p-adic Hurwitz zeta function. A novel aspect of our approach is the generalization of Beukers' method and the construction of a Padé approximation for the p-adic polygamma functions.

15:50 – 16:30 **Masatoshi Suzuki (Institute of Science Tokyo)**

"On M-functions and screw functions originating from Goldbach's problem "
(joint work with Kohji Matsumoto)

We study the M-functions, which describe the limit theorem for the value-distributions of the secondary main terms in the asymptotic formulas for the summatory functions of the Goldbach counting function. One of the new aspects is a sufficient condition for the Riemann hypothesis provided by some formulas of the M-functions, which was a necessary condition in previous work. The other new aspect is the relationship between the secondary main terms and the screw functions, which provides another necessary and sufficient condition for the Riemann hypothesis.